A 43-level filterless CMLI with very low harmonics values

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Abstract

This paper introduces a 43-level asymmetric uniform step cascaded multilevel inverter (CMLI) that consists of four H-bridges per phase, with different dc sources of values E, 2E, 7E and 11E. A mixed integer linear programming (MILP) optimization model is applied to determine the switching angles of the CMLI power switches that can minimize the values of any undesired harmonics. Single phase and three phase cases are considered. The results show very low values of all the undesired harmonics over wide voltage ranges, which agree with the IEEE standards 519-1992 for voltage distortion limits for both the values of %THD_E and %V_{lim, ES} so that no output filters are needed.

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1. Introduction

The concept of multilevel inverter, introduced about 35 years ago (Nabe et al., 1980), entails power conversion in multiple voltage steps to obtain improved power quality, lower switching losses, better electromagnetic compatibility and higher voltage capability. Among the existing multilevel inverter topologies, the cascaded multilevel inverter (CMLI) reaches the higher voltage and power levels and higher reliability due to its modular topology and simplicity (Leon et al., 2009). Recently, asymmetric CMLIs have received special attention (Sivagamasundari and Mary, 2014; Sundari and Dhiankaraj, 2014; Chandwani et al., 2013; Khadtare et al., 2013; Banaei and Salary, 2013; Seyezhai, 2012). They utilize small number of H-bridges with different DC source values to obtain huge number of output voltage steps, so that the output voltage waveform can approach a pure sine waveform and consequently the output voltage undesired harmonics could be reduced effectively.

Fig. 1 shows a general single phase asymmetric CMLI, that consists of S H-bridges with DC sources of different values. The output voltage waveform usually takes a staircase shape, to approach the shape of a sine wave. The asymmetric CMLI may have uniform step or non-uniform step output waveform (El-Bakry, 2012). Fig. 2 shows, as a simple example, the output waveform of a 7-level uniform step asymmetric CMLI with two H-bridges having the
DC sources $V_{dc}$ and $\frac{1}{2}V_{dc}$ (Seyezhai, 2011b), and producing three positive levels, where the voltage levels are equally spaced. The main challenge for general CMLIs is to determine the switching angles $\theta_1$, $\theta_2$, etc. from one level to the next level that eliminate or minimize as possible the values of undesired harmonics, especially when the number of levels increases greatly, to approach the shape of a pure output sine waveform.

This paper introduces a 43-level uniform step asymmetric CMLI, which is realized by four H-bridges with DC sources of values $E$, $2E$, $7E$ and $11E$. This CMLI can produce 21 positive voltage levels, by switching the DC sources

Fig. 1. A general single phase asymmetric CMLI.

Fig. 2. A 7-level staircase output voltage waveform.
E and 2E positively or negatively with the DC sources 7E and 11E during the positive quarter time cycle of the output voltage to obtain the voltage levels E, 2E, 3E, . . . , 21E. To obtain the switching angles for the H-bridges of this CMLI, many methods are proposed in the literature, the most popular of them are:

1. Using selective harmonic elimination technique. Equations of the harmonics in the output voltage waveform are obtained by Fourier expansion as functions of the switching angles \( \theta_1, \theta_2 \), etc. The zero equations of the undesired harmonics are solved with the equation of the desired amplitude of the main harmonic to obtain the values of the switching angles (Napoles et al., 2013; Ahmadi et al., 2011; Kumar et al., 2009). For the 43-level CMLI it will be required to solve 21 simultaneous equations to obtain the values of the 21 switching angles in the quarter positive cycle of the waveform, and these are very difficult to solve, and would be probably subject to non feasible solution due to the trigonometric nature of the equations, even if genetic algorithms are applied (Filho et al., 2013; Kavousi et al., 2012).

2. Using pulse width modulation (PWM) techniques (Sasi et al., 2013; Kiruthika et al., 2013; Mohan and Kurub, 2012; Seyezhai, 2011a, 2012). This would be difficult to realize practically with huge number of levels.

3. Using methods for minimizing the total harmonic distortion (Shahipour et al., 2014; Yousefpoor et al., 2012; Karthiyan and Pandian, 2011).

However, all these methods are not suitable for CMLIs with large number of levels.

Most of these methods would need output filters to reduce high order harmonics, even if low harmonics are reduced or rejected.

In addition, the author has introduced a method based on a general linear programming model that could be applied to minimize the values of the undesired harmonics (El-Bakry, 2009, 2010). This linear programming (LP) model has the following advantages over other methods discussed in the literature (El-Bakry, 2013):

1. This model is flexible. It allows minimizing the values of harmonics of any order and any number, independent of the number of inverter levels, which may be very large, under any required reasonable amplitude of the main harmonic.

2. LP constraints may be inequality constraints, so a feasible solution could be almost found, unlike harmonic elimination methods that depend on equalities and may give no feasible solution due to not satisfying the trigonometric equalities imposed.

3. LP constraints allow minimizing harmonics values with different weighting factors according to the harmonic order. High order harmonics could be minimized as well as low order harmonics and low order harmonics values could be minimized much more than higher order ones.

4. LP provides global optimal solution of the problem over the whole solution space, unlike some other optimization methods that give a local optimal solution near an initial solution, which may not be a global optimal solution.

5. Many software packages are available for solving LP models, even with huge number of variables and constraints, in a moderate time. They are suitable for problems with huge number of levels that could not be easily solved with other methods.

6. The model can be applied even for asymmetric CMLIs with non uniform steps by adding additional constraints (El-Bakry, 2012).

7. The model contains many parameters that could be selected arbitrarily. Many optimum solutions could be obtained for the same problem corresponding to different values of these parameters, and thus allowing for selecting the best one.

The mathematical model of this approach is first given, and then applied for the single phase and the three phase 43-level CMLIs. The model is applied first to determine the number of harmonics to be minimized that lead to least THD. By including this result in the model and solving it for different amplitudes of the output main harmonic, the switching patterns that give minimum values of undesired harmonics could be obtained.
2. The proposed mathematical model

The general uniform step asymmetric CMLI, or symmetric CMLI, is considered, where all the inverter levels are spaced equally with a step height $E$. It is assumed, without loss of generality, that the inverter levels are equally spaced by $1V$, i.e. normalized with respect to the dc voltage $E$. It is assumed also that the inverter output voltage waveform $F(wt)$ has a quarter wave symmetry, as that shown in Fig. 2. The pattern of this function is generated by on and off switching of the inverter H-bridges power switches, and is completely determined by defining the switching pattern over the interval $0 \leq wt \leq \pi/2$. The basic approach depends on dividing this interval into $N$ equal small subintervals, starting at the angles $0, \tau, 2\tau, \ldots, (I-1)\tau, \ldots, (N-1)\tau$, where $\tau = \pi/2N$ (Fig. 3).

The positive integer values $X_I, I = 1, 2, \ldots, N$ are defined over each subinterval, to represent the required instantaneous output voltage level value $F(wt)$ of the inverter, so that $F(wt)$ is defined over the interval $0 \leq wt \leq \pi/2$ by:

$$F(wt) = X_I \quad \text{for} \quad (I - 1)\tau \leq wt \leq I\tau \quad \text{and} \quad I = 1, 2, \ldots, N$$

The Fourier series expansion of $F(wt)$ is an odd-sine series given by:

$$F(wt) = \sum_{m=0}^{m=\infty} V_{2m+1} \sin(2m + 1)wt,$$

where

$$V_{2m+1} = (4/\pi) \int_{0}^{\pi/2} F(wt) \sin(2m + 1)wt \, dw = (8/\pi(2m + 1)) \sum_{I=1}^{I=N} X_I \sin(2m + 1)(\tau/2)\sin(2m + 1)(\Phi_I + \tau/2)$$

(1)

where $(2m + 1)$ is the order of the harmonic, $m = 0, 1, 2, \ldots, \infty$, $\tau = \pi/2N$, and $\Phi_I = (I - 1)\tau$.

The value of the amplitude of main harmonic corresponds to $V_1$, and is obtained by substituting $m = 0$ in Eq. (1).

Eq. (1) shows that $V_{2m+1}$ for any value of $m$ is a linear function of the integer values $X_I, I = 1, 2, \ldots, N$.

Variations of the values of $X_I$ from a subinterval to a next one determine the required switching angles of the inverter from one level to another.

It is required to find the values of $X_I$ that minimize the values of some undesired harmonics. A mixed integer linear programming (MILP) problem is formulated as follows (El-Bakry, 2010):

Minimize $\epsilon$, subject to the constraints:

$$V'_1 - \Delta \leq V_1 \leq V'_1 + \Delta \quad \text{(2)}$$

$$-\epsilon\alpha_{2m+1} \leq V_{2m+1} \leq \epsilon\alpha_{2m+1}, \quad \text{for each undesired harmonic of order} \quad (2m + 1) \quad \text{(3)}$$

$$X_I \leq X_{I+1}, \quad \text{for} \quad I = 1, 2, \ldots, N - 1, \quad \text{and} \quad X_N \leq L \quad \text{(4)}$$

$$X_I \geq 0 \quad \text{and integer for} \quad I = 1, 2, \ldots, N \quad \text{(5)}$$

In the main harmonic constraint (2) $V'_1$ is the required amplitude of the main harmonic, $\Delta$ is a small incremental value, $\Delta \ll V'_1$, arbitrary chosen and included in the main harmonic constrain to ensure obtaining an optimum solution, since
an equality constraint may give a high value of $\varepsilon$ or even an unfeasible solution, due to the trigonometric nature of the constraints. The value of $\Delta$ is taken of the order of 1% of $V_1'$, so that the obtained value of $V_1$ does not differ practically from the required value of $V_1'$.

In constraint (3) $V_{2m+1}$ is given by Eq. (1), for the undesired harmonics, and $\alpha_{2m+1}$ is a weighting factor for the undesired harmonics, to enable reduction of harmonics with different upper bounds according to their order.

By constraints (4) the positive staircase waveform shape is assured with maximum height $L$, where $L$ is the number of positive voltage levels of the inverter.

Constraints (5) are the integer constraints on $X_I$.

Once all the parameters of this MILP model are given, an optimum solution could be obtained that gives the values of $X_I$ and $\varepsilon$ using any of the well known operations research software packages, e.g. “LINGO” software (Schrage, 2013).

When solving this model, it will include, in addition, formulas for calculating the exact total harmonic distortion (THDE) and the upper limit of the amplitude of any undesired harmonic relative to the amplitude of the main harmonic (%$V_{\text{Hmax}}$), which are given in the next section.

3. Formulas for calculating %THDE and %$V_{\text{Hmax}}$ of the undesired output harmonics

3.1. Formulas for calculating the %THDE

The exact total harmonic distortion (%THDE), calculated for all possible undesired harmonics is given by:

$$\text{THDE} = \left[ \sum_{m=1}^{m=\infty} \left( \frac{V_{2m+1}}{V_1} \right) \right]^{0.5}$$

(6)

The value of THDE could be obtained from the solution of the MLIP model, after obtaining the values of $X_I$ using the following expressions (El-Bakry, 2014):

- For single phase CMLI, the output phase voltage THDE is given by:

$$\text{THDE} = \left[ \left\{ (1/N) \sum_{l=1}^{l=N} \frac{X_l^2}{V_{1\text{rms}}} \right\} - 1 \right]^{0.5}$$

(7)

- For a balanced three phase CMLI, the output line voltage THDE is given by, assuming that $N$ is a multiple integer of 3:

$$\text{THDE} = \left[ \left\{ (1/2N) \sum_{l=1}^{l=2N} \frac{Z_l^2}{V_{2\text{rms}}} \right\} - 1 \right]^{0.5}$$

(8)

where:

$$Z_I = X_I - Y_I \quad \text{for} \quad I = 1, 2, \ldots, N$$

$$Z_I = XX_I - YY_I \quad \text{for} \quad I = N + 1, \ldots, 2N$$

$$Y_I = -X_{(2N/3)+1} \quad \text{for} \quad I = 1, 2, \ldots, N/3$$
\[ Y_I = -X_{(4N/3)-1} \quad \text{for} \quad I = (N/3) + 1, \ldots, N \]

\[ XX_I = X_{2N+I-1} \quad \text{for} \quad I = N + 1, \ldots, 2N \]

\[ YY_I = -X_{(4N/3)+I-1} \quad \text{for} \quad I = N + 1, \ldots, 4N/3 \]

\[ YY_I = X_{I-(4N/3)} \quad \text{for} \quad I = (4N/3) + 1, \ldots, 2N \]

3.2. The formula for calculating the %\( V_{H_{\text{max}}} \)

To calculate the upper limit of the amplitude of any harmonic among all the undesired harmonics relative to the amplitude of the main harmonic (%\( V_{\text{LH}} \)), the MILP model is programmed to calculate the amplitude of the harmonics \( V_{2m+1} \) till the 91st harmonic, i.e. for \( m = 0, 1, 2, \ldots, 45 \) then the following value are calculated:

1. The maximum amplitude value among all the undesired low order harmonics till the 91st harmonic (%\( V_{\text{LH}} \)) relative to the amplitude value of the main harmonic:

\[ V_{\text{LH}} = \max_{m=1,\ldots,45}(V_{2m+1}/V_1) \]  \hspace{1cm} (9)

2. The total harmonic distortion (%\( \text{THD}_{91} \)) of the low order harmonics calculated till the 91st harmonic:

\[ \text{THD}_{91} = \left[ \sum_{m=1}^{45} \left( \frac{V_{2m+1}}{V_1} \right)^2 \right]^{0.5} \]  \hspace{1cm} (10)

3. The rout of the sum of squares of the amplitudes of all the high order harmonics above the 91st harmonic relative to the square of the amplitude of the main harmonic (%\( V_{\text{HH}} \)) is calculated according to the expression (El-Bakry, 2014):

\[ V_{\text{HH}} = [\text{THDE}^2 - \text{THD}_{91}^2]^{0.5} \]  \hspace{1cm} (11)

This value could be taken as an upper bound of the amplitude of any harmonic among all the undesired harmonics above the 91st harmonic relative to the amplitude of the main harmonic.

From (9) and (11) the upper limit of the amplitude of any harmonic among all the undesired harmonics relative to the amplitude of the main harmonic (%\( V_{H_{\text{max}}} \) over the whole frequency band could be taken as:

\[ V_{H_{\text{max}}} = \max(V_{\text{LH}}, V_{\text{HH}}) \]  \hspace{1cm} (12)

In the next sections when the MILP model is applied, the values of the %\( \text{THDE} \) and the %\( V_{H_{\text{max}}} \) are calculated and compared with the IEEE standards 519-1992 for voltage distortion limits for both the %\( \text{THDE} \) and the %\( V_{H_{\text{max}}} \) (IEEE Standard 519-1992), which put the following limits:

For output voltages \( \leq 69 \text{kV} \), the %\( \text{THDE} \) must be \( \leq 5\% \) and the %\( V_{H_{\text{max}}} \) must be \( \leq 3\% \).

For output voltages between 69 kV and 161 kV, the %\( \text{THDE} \) must be \( \leq 2.5\% \) and the %\( V_{H_{\text{max}}} \) must be \( \leq 1.5\% \).
In the following sections, this model is applied for the 43-level single phase and three phase CMLIs, taking the number of subintervals $N = 720$, that corresponds to a subinterval angular width $\tau = 90^\circ/720 = 0.125^\circ$ (Fig. 3). The model is solved first with the constraint (2) replaced by:

$$V_1 \geq L$$

(13)
to minimize undesired harmonics for all amplitudes of the output voltage greater than $L$, normalized w.r.t. $E$, while utilizing all the levels of the inverter ($L = 21$). The undesired harmonics are minimized equally till a harmonic of order $k$, for different values of $k$, then selecting the value of $k$ that leads to least $\%$THDE. This value of $k$ is included in the model, and the model is solved to obtain the switching angles for different values of the output voltage. From the resulting solutions, the values of $\%$THDE and $\%V_{H\text{max}}$ obtained are compared with the required IEEE standards 519-1992 for voltage distortion limits in electrical power systems (IEEE Standard 519-1992).

4. Solution of the MILP model for the 43-level single phase CMLI

4.1. Solution for different values of the undesired harmonics

Fig. 4 shows the values of $\%$THD$_{91}$ and $\%$THDE obtained by solving the MILP model with the voltage constraint (13) replacing the constraint (2) to minimize the odd harmonics 3, 5, etc. till the 4th harmonic, for different value of $k$. The least $\%$THDE is obtained by minimizing the undesired harmonics equally till the 49th harmonic ($k = 49$), and this is obtained at $V_1 = 21.19$.

4.2. Solution for different values of the output voltage

The model is solved to minimize the undesired low order harmonics till the 49th harmonic using the constraint (2) for some values of $V'_1$ between 9 and 22, and taking $\Delta = 0.2$. Fig. 5 shows the obtained values of $\%$THD$_{91}$, $\%$THDE,
%\(V_{HH}\) and \(V_{LH}\). At \(V_I = 21.19\), these values are 1.58, 2.12, 1.41 and 0.61 respectively, thus the %THDE = 2.12 and according to Eq. (12) \(V_{Hmax} = \max(1.41, 0.61) = 1.41\), and this satisfies the IEEE standards 519-1992 for voltage distortion limits till 161 kV. While for other voltage amplitudes between 11 and 22 the %THDE and \(V_{Hmax}\) are \(\leq 5\%\) and \(\leq 3\%\) respectively, which satisfies the IEEE standards 519-1992 for voltage distortion limits till 59 kV.

The detailed solution of the model for \(V_I = 21.19\) is given next. Fig. 6 shows the obtained values of \(X_I\). The 21 switching angles of the inverter from the zero level to the 21st level during the quarter positive cycle of the output voltage are \(11\tau, 19\tau, 51\tau, 78\tau, 89\tau, 123\tau, 140\tau, 165\tau, 191\tau, 208\tau, 240\tau, 261\tau, 294\tau, 313\tau, 348\tau, 377\tau, 412\tau, 446\tau, 488\tau, 535\tau, \) and \(599\tau\) respectively, where \(\tau = 0.125^\circ\). These levels may be achieved during the quarter positive cycle of the output voltage by the switching patterns of the four H-bridges of the CMLI shown in Fig. 7. The 11E H-bridge is switched on only once, the 7E H-bridge is switched on two times and switched off one time, the 2E H-bridge is switched on positively four times and switched off three times and switched negatively on and off two times, while the 1E H-bridge is switched on positively eight times and switched off seven times and switched negatively on and off four times. The switching losses of the 1E and 2E H-bridges are less than that of the higher voltage 7E and 11E H-bridges (Faraneh and Nazarzadeh, 2009), so the switching losses are kept at least values. Generally, the switching losses do not represent a serious problem with modern development of semiconductor power switches with low switching losses (Sheklowat and Brockway, 2009).

Fig. 8 shows the obtained percentage amplitudes of the undesired harmonics relative to the amplitude of the main harmonic till the 91st harmonic and 2% of the amplitude of the main harmonic for \(V_I = 21.19\).

5. Solution of the MILP model for the 43-level three phase CMLI

In a balanced three phase system the triplen odd harmonics are self cancelled in the line voltage. The values of the amplitudes of the other undesired harmonics relative to the main harmonic in the line voltage are equal to that in
The phase voltage. The procedures carried out in Section 4 for single phase 43-level CMLI are repeated next while excluding the triplen odd harmonics and using Eq. (8) for calculating the %THDE.

5.1. Solution for different values of the undesired harmonics

Fig. 9 shows the values of %THD$_{91}$ and %THDE obtained by solving the MILP model with the voltage constraint (13) replacing the constraint (2) to minimize the odd harmonics 3, 5, etc. till the 4th harmonic, for different value of $k$. The least %THDE is obtained by minimizing the undesired harmonics equally till the 55th harmonic ($k = 55$), and this is obtained at $V_1 = 21.95$.

5.2. Solution for different values of the output voltage

The model is solved to minimize the undesired low order harmonics till the 55th harmonic using the constraint (2) for some values of $V'_1$ between 9 and 22, and taking $\Delta = 0.2$. Fig. 10 shows the obtained values of %THD$_{91}$, %THDE, %$V_{HH}$ and %$V_{LH}$. For output voltage amplitudes between $V_1 = 15$ and 24 the %THDE is less than 2.50% and according to Eq. (12) the %$V_{H_{\text{max}}}$ is less than 1.5%, and this satisfies the IEEE standards 519-1992 for voltage distortion limits till 161 kV.

The detailed solution of the model for $V_1 = 21.95$ is given next. The obtained values of %THD$_{91}$, %THDE, %$V_{HH}$ and %$V_{LH}$ are 0.92, 1.36, 1.00 and 0.51 respectively. Fig. 11 shows the obtained values of $X_i$. The 21 switching angles of the inverter from the zero level to the 21st level during the quarter positive cycle of the output voltage are $8\tau, 18\tau, 33\tau, 61\tau, 85\tau, 118\tau, 124\tau, 154\tau, 163\tau, 189\tau, 207\tau, 231\tau, 255\tau, 281\tau, 313\tau, 350\tau, 384\tau, 405\tau, 433\tau, 535\tau$, and $571\tau$ respectively, where $\tau = 0.125^\circ$. These levels may be achieved during the quarter positive cycle of the output voltage by the switching patterns of the four H-bridges of the CMLI in a way similar to that shown in Fig. 9. The 11E H-bridge is switched on only once, the 7E H-bridge is switched on two times and switched off one time, the other two H-bridges are switched on and off positively and negatively multiple times.
Fig. 10. The values of $\%\text{THD}_{91}$, $\%\text{THDE}$, $\%V_{HH}$ and $\%V_{LH}$ for different values of $V_1$.

Fig. 11. Values of $X(t)$ that give $V_1 = 21.95$.

Fig. 12. Values of undesired harmonics for $V_1 = 21.95$.

Fig. 12 shows the obtained percentage amplitudes of the undesired harmonics relative to the amplitude of the main harmonic till the 91st harmonic and 2% of the amplitude of the main harmonic for $V_1 = 21.95$.

6. Conclusions

This paper introduces a 43-level uniform step asymmetric cascaded multilevel inverter (CMLI) that consists of four H-bridges that can give output voltages with very low values of the undesired harmonics for both single phase and three phase applications. A mixed integer linear programming model is applied to determine the switching angles of the semiconductor power switches of the inverter that minimize the values of any undesired harmonics, and has many advantages over other methods given in the literature. This model is applied first to determine the number of harmonics to be minimized for least $\%\text{THDE}$ when utilizing all the CMLI levels. Then these values are used when solving the model for different values of the output voltage. The results show very low values of all the undesired harmonics over
wide voltage ranges, which agree with the IEEE standards 519-1992 for voltage distortion limits for both the values of \%THDE and \%V\textsubscript{Hmax}, for single phase CMLI till output voltage ≤69 kV and three phase CMLI till output voltage ≤161 kV and no output filters are needed. Due to the very low values of the undesired output harmonics, the losses in these harmonics are very low and the efficiency of the CMLI is high, and is mainly determined by the switching and conduction losses of the power switches of the CMLI, and these losses will tend to decrease with the continuous development in power switches.

The proposed CMLI could be used for many applications under different power ranges such as for electric drives, electric vehicles and power systems.

References


