Journal of Traffic and Transportation Engineering (English Edition)
2014,1(6):383-392

Operational characteristics of mixed traffic flow under bi-directional environment using cellular automaton

Zhenke Luo[1], Yue Liu[1,*], Chen Guo[2]

[1]*Department of Civil and Environmental Engineering, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin, USA*

[2]*School of Control Science and Engineering, Shandong University, Jinan, Shandong, China*

Abstract: Mixed traffic flow composed of autos and non-autos widely exists in developing countries and areas. To investigate the operational characteristics of the mixed traffic flow consisting of vehicles in different types (large vehicles, cars, and bicycles), we develop a cellular automaton model to replicate the travel behaviors on a bi-directional road segment with respect to the physical and mechanic features of different vehicle types. By implementing the essential parameters calibrated through the field data collection, a numerical study is carried out considering the variation in volume, density, and velocity with different compositions of mixed traffic flows. The primary findings include: the average velocity of traffic flow and total volume decrease 60% and 30% after incorporating 10% bicycles, respectively; the phenomenon of double-summit in terms of the total volume appears when the proportion of bicycle is beyond 60%; the maximal total volume starts to recover when the proportion of bicycle is higher than 10%.

Key words: mixed traffic flow; operational characteristic; cellular automaton; bi-directional environment

1 Introduction

1.1 Background

Mixed traffic flow consisting of autos and non-autos widely exists in many developing countries and areas, such as China, India and Indonesia (Khan and Maini 1999), especially at road segments without median separation. The discrepancies in the operational characteristics of different vehicles and their interactions and interferences play an important role in affecting the traffic operational efficiencies (e. g. throughput, speed, volume, etc.). To capture and replicate such behaviors, many researchers have made attempts on developing various types of methods, tools, and models to better understand such operational characteristics.

In review of the literature, early efforts tackling

* **Corresponding author**: Yue Liu, PhD, Associate Professor.
E-mail: liu28@uwm.edu.

with traffic flow modeling primarily apply statistical methods to explain the fundamental relations between flow, density and speed. One pioneering work illustrated the potential of applying Poisson distribution in explaining traffic operational characteristics (Kinzer 1993). Adams published the statistical result considering the input of traffic flow as random series. Greenshields et al. (1947) used Poisson distribution in investigating the traffic flow across intersections.

In the 1950s-1970s, more researchers have developed advanced models gaining a better description of traffic flow instead of performing statistical analysis in early research. For example, the car following theory (Chander et al. 1958; Herman et al. 1959) focused on interactions between the leading vehicles and following vehicles. Lighthill and Whitham (1955) demonstrated that the moving of traffic flow is similar to other phenomena in the natural world such as ocean waves, avalanches, and debris flows. By incorporating the fluid mechanic theory, FREFLO (Payne 1971, 1979) was developed and widely used in real world practices.

Recently, the advance of computational technology has facilitated the exploration of more sophisticated microscopic traffic flow models that are able to successfully capture the behaviors of individual vehicles and pedestrians with respect to various influential factors, such as types of vehicles, weathers, facility types, and control methods. Cellular automaton (CA) is one of the most prevailing and successful microscopic models. It was first applied in the transportation field to simulate car movements including lane changing, turning, queuing, acceleration, and deceleration in the road network, and the results demonstrated its ability to capture the phenomena of macroscopic models while in the meantime reproduce the mechanics of microscopic models (Cremer and Ludwig 1986). Nagel and Schreckenberg (1992) extended the cellular automaton model by setting more traffic evolvement rules to capture more realities. Fukui and Ishibashi (1996) considered that the operational speed of an individual vehicle is not only influenced by its leading vehicle, but also by the density of the neighboring environment. Foulaadvand and Belbasi (2007) studied vehicular traffic flow at an un-signalized intersection by a cellular automaton model and validated the model characteristics by a mean-field approach and extensive simulations. Ruskin and Wang (2007) studied un-signalized intersections by introducing the concept of acceptable headway.

In modeling mixed traffic flow using cellular automaton, Gundaliya et al. (2008) developed the models with multiple cell occupancy, reflecting sizes and shapes of different vehicles to reproduce the macroscopic properties of heterogeneous traffic typical of Indian cities. Jiang et al. (2004) modelled the "in bulk" movement of traffic flow consisting of bicycles. Whereas, Vasic and Ruskin (2012) modeled the mixed traffic flow in which bicycles sparsely spread in a one dimensional single lane environment. Meng et al. (2007) incorporated motorcycles into the traffic to investigate the interrelation between different vehicle types and the impact on traffic operations. Xie et al. (2009) used CA model to investigate mixed traffic flows at un-signalized intersections and suggested that the velocity difference between different types of vehicles is an important factor reflecting travel behaviors. Zhao and Gao (2005) described mixed traffic flow by combining the NaSch model (Nagel and Schreckenberg 1992) and the Burger cellular automata (BCA) model (Nishinari and Takahashi 1998), and investigated the mixed traffic system near a bus stop.

The problem to be addressed in this paper is the lack of a comprehensive model describing the traffic condition associated with mixed traffic flow at bi-direction road segments. Under a bi-directional environment without median separation, the under estimation of the impacts caused by the opposing flow at the other lane and the differences underlying lane-changing behavior will limit the model's applicability and result in simulation results far away from the reality.

1.2 Research objectives

To contend with the above problems, the objectives of this study are to investigate the operational characteristics of mixed traffic flow consisting of autos and bicycles at bi-directional road segments without the setting of exclusive lanes and road medians. More specifically, we will determine the representation of mixed traffic flow in a two-lane bi-directional environment; develop the evolvement rules for the CA model considering the physical and mechanic characteristics of different types of vehicles under a bi-directional environment; investigate the operational characteristics of mixed traffic flow by using the proposed model.

2 Model development

2.1 Basic CA model

Our model is based on the NaSch model which is defined on a one-dimensional array of L "cells" under open or periodic boundary conditions and each "cell" may either be occupied by at most one vehicle or be empty as shown in Fig. 1. Suppose x_n and v_n denote the location and the speed of vehicle n, respectively, and $v_{\max}$ represents the maximum velocity of a vehicle. $d_n = x_{n+1} - x_n - 1$ is the distance from the vehicle n to its front vehicle. Then each vehicle can move with an integer velocity. An update of vehicle state in the CA model involves with the following four consecutive steps.

Step 1: Acceleration, if $v_n < v_{\max}$, then increase the velocity of vehicle n by one unit, $v_n = \min\{v_n + 1, v_{\max}\}$.

Step 2: Deceleration, if $d_n < v_n$, then the speed of vehicle n is decreased to d_n, i. e. $v_n = \min\{v_n, d_n\}$.

Step 3: Randomization, if $v_n > 0$, then the velocity decreases by one unit with a probability p which is accounting for conditions that the velocity decreases due to the influence of other uncertain factors, such as pedestrians, obstructions, and distractions, i. e. $v_n = \max\{v_n - 1, 0\}$.

Step 4: Movement, the vehicle updates its location with the velocity determined by Steps 1-3, i. e. $x_n = x_n + v_n$.

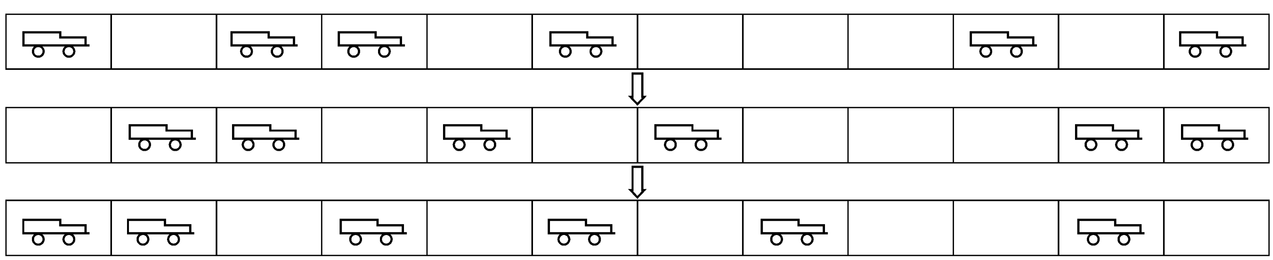

Fig. 1 Illustration of NaSch model

2.2 Cell size specification

To establish our model, we need to make some additional assumptions to determine the size of the "cell" aiming at achieving a convenient representation of the operations of different vehicle types in the mixed traffic flow.

Firstly, given a typical two-lane urban road with a 3-meter lane width, we need to know how many bicycles can be accommodated. According to Ren et al. (2003), we assume that one single lane can hold 3 bicycles in a row and a bicycle may share a cell with a car. A large vehicle (e. g. bus, truck) is assumed to occupy 3 m according to our observation. For the longitude space required for different types of vehicles, a bicycle is assumed to occupy a 2.75 m longitude distance in a complete traffic jam, which is the half of the distance that a car needs. While the length for a large vehicle considering the safety distance is set to be 11 m.

With the information regarding the length and width for each type of vehicle, we can determine the cell size to be 2.75 m by 3.00 m, which is able to accommodate 3 bicycles. A car takes 2 cells in length and 2/3 cell in width. While a large vehicle takes 4 cells in length, 1 cell in width. The similar concept has already been employed in the case of mixed car/truck traffic in which cars are "shorter" vehicles and trucks are "longer" vehicles (Nagel and Schreckenberg 1992). An example of mixed traffic flow representation environment at a two-lane bi-directional road segment is illustrated in Fig. 2.

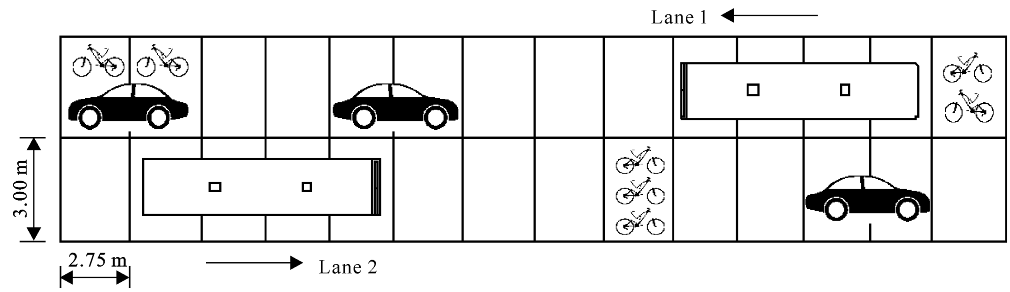

Fig. 2 Representation of mixed traffic flow at a two-lane bi-directional road segment

2.3 Evolvement rules

The evolvement rules set for the mixed traffic flow should take into account the mechanical characteristics of different types of vehicles. For example, bicycles are smaller in size and easier to operate, therefore they have more flexibility in acceleration, deceleration, and lane changing. However, they are associated with a relative low maximal velocity compared with other vehicles. Cars have a higher maximal velocity but a lower probability for conducting lane-changing behaviors due to the limitation in their sizes and safety considerations. In our model, car following, lane changing, and parallel operating are explicitly modeled and illustrated in Fig. 3.

For the lane changing behavior, there should be sufficient gap in the target lane with the consideration of safety. Technically, there must be sufficient intervals to meet the requirements that

$$d_{i,n}^{\mathrm{B}} \geqslant gap_i^{\mathrm{B}}$$

$$d_{i,n}^{\mathrm{opp}} \geqslant gap_i^{\mathrm{opp}}$$

where $d_{i,n}^{\mathrm{B}}$ is the distance from the back of the vehicle n in type i to the back of the vehicle that has just passed it on the target lane; $d_{i,n}^{\mathrm{opp}}$ is the gap from the front of the vehicle to the front of the closest one on the other lane; gap_i^{B} and gap_i^{opp} account for the minimal back and front gaps for vehicle type i ensuring the safety of lane changing behavior, respectively (Fig. 3).

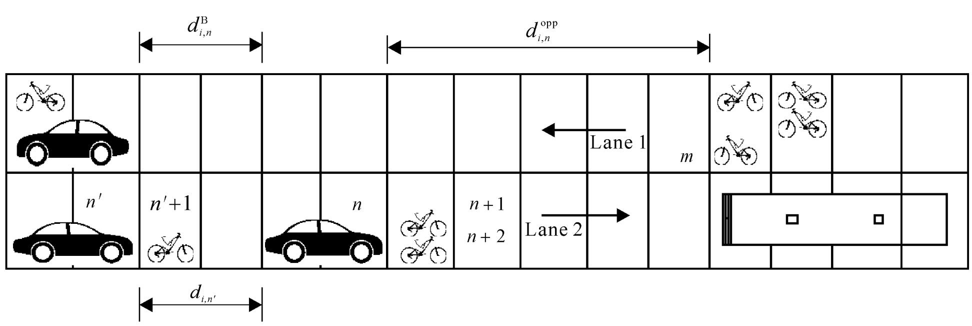

Fig. 3 Illustration of driving behaviors

The second criterion for lane changing is that the operational velocity of the front vehicle is slower than the vehicle's expected velocity in the next time step, and the driver expects a better driving condition on the other lane. Such situation becomes more complicated when there are multiple vehicles ahead (e. g. two bicycles in front of the vehicle n in Fig. 3). To account for such situation, we deem that the speed criterion is satisfied if the minimal expected speed of the front vehicles is lower than the expected velocity of current vehicle, given by

$$\min\{v_{i,\max}, v_{i,n}+1\} > \min\{v_{j,n+1}+1, v_{j,\max}, v_{k,n+2}+1, v_{k,\max}\} \quad \forall d_{n,n+1} = d_{n,n+2}$$

where $d_{n,n+1}$ and $d_{n,n+2}$ represent the distances from vehicles $n+1$ and $n+2$ to vehicle n, respectively; $v_{i,\max}$ is the maximal velocity set for the vehicle type i.

Since the proposed model allows multiple vehicles operating in parallel, it is possible that a vehicle can overtake another one without lane changing. For example in Fig. 3, the distance from the vehicle n to its front vehicle (i. e. $d_{i,n'}$) is equal to 2 instead of 0 because there is sufficient space in the cell occupied by the vehicle $n'+1$. The same criterion also applies for the calculation of $d_{i,n}^{opp}$ and $d_{i,n}^{B}$. Furthermore, we adopt the concept of positional discipline which implies that in a mixed traffic flow environment, bicycles keep to the side of the road nearest to the curb, while autos accommodate space for any present bicycles by staying as far away from the curb as possible without crossing the median line.

Considering safety, we also assume that when the space is sufficient ($d_{i,n}^{opp} \geqslant 0$), the vehicle making a change to the opposing lane will immediately go back to the farthest location on the original lane with respect to its velocity. If there is no space allowing vehicles changing to the opposing lane to move back ($d_{i,n}^{opp} < 0$, e.g. the vehicle n in Fig. 3), the vehicle will keep driving on the opposing lane until there is a gap.

Although most CA studies have modeled single-lane (Nagel and Schreckenberg 1992; Fukui and Ishibashi 1996; Vasic and Ruskin 2012) or multiple-lanes (Nishinari and Takahashi 1998; Jiang et al. 2004; Meng et al. 2007) operations in one direction and have incorporated speed randomization factors to reflect the changes in speed due to uncertainties, it might not be sufficient to represent the impact of opposing flows under a bi-directional environment. As drivers may be more cautious and will decrease the speed with a higher probability to avoid the conflict with the opposing flows, we propose to set the randomization factor for type i vehicles that are experiencing or have just experienced a crossing on the opposing lane to be p_i'. For those vehicles that are not facing crossings, their randomization factor is set to be p_i. However, this rule is not imposed on vehicles driving on the opposing lane when they are overtaking because of no conflict between the flows on the original lane.

In summary, the evolvement rules in the proposed model are given by

Step 1: Acceleration

$$v_{i,n} = \min\{v_{i,n}+1,\ v_{i,\max}\} \quad \forall v_{i,n} \in \{0,1,2,\cdots,v_{i,\max}\}$$

For the vehicle n in the opposing lane

$$v_{i,n}^{opp} = \min\{v_{i,n}^{opp}+1,\ v_{i,\max}\} \quad \forall v_{i,n}^{opp} \in \{0,1,2,\cdots,v_{i,\max}\}$$

Step 2: Deceleration

$$v_n = \min\{v_n,\ d_n\} \quad \forall d_n \in \{0,1,2,\cdots,N\}$$

For the vehicle n in the opposing lane

$$v_{i,n}^{opp} = \min\{v_{i,n}^{opp},\ d_n\}$$

Step 3: Randomization

$$v_{i,n} = \max\{v_{i,n}-1,\ 0\}$$

with the probability p_i' for the vehicle n experiencing or experienced a crossing on the opposing lane, otherwise with a probability p_i.

For the vehicle n in the opposing lane

$$v_{i,n}^{opp} = \max\{v_{i,n}^{opp}-1,\ 0\}$$

with a probability p_i.

Step 4: Lane changing

For the vehicle n in the original lane, if

$$\min\{v_{i,\max}, v_{i,n}+1\} > \min\{v_{j,n+1}+1, v_{j,\max}, v_{k,n+2}+1, v_{k,\max}, \cdots\} \quad \forall d_{n,n+1} = d_{n,n+2} = \cdots$$

$$d_{i,n}^{B} \geqslant gap_i^{B}$$

$$d_{i,n}^{opp} \geqslant gap_i^{opp}$$

then, the vehicle changes the lane with a probability p_i^{opp} and $v_{i,n} = v_{i,n}^{opp}$.

For the vehicles on the opposing lane

$$v_{i,n} = v_{i,n}^{opp} = \min\{d_{i,n}^{opp},\ v_{i,n}^{opp}\} \quad \forall d_{i,n}^{opp} \geqslant 0$$

Step 5: Vehicle movement

For the vehicle n that operates or comes back to the original lane

$$x_n = x_n + v_{i,n}$$

For the vehicle n on the opposing lane

$$x_n = x_n + v_{i,n}^{opp}$$

where $v_{i,n}^{opp}$ indicates that the velocity of the vehicle n of type i on the opposing lane.

3 Model calibration and validation

This section details the calibration of the essential model parameters and sets the following criteria for data collection site selection:

1) Low impact of pedestrians;

2) Low impact of signalized intersections: intersection signals are not explicitly modeled in our study, thus the data collection site should be far away from signals;

3) Low impact of roadway parking.

According to the above requirements, we performed data collection during the peak periods (7 am-9 am and 5 pm-7 pm) on August 27, 2009 at a two-lane bi-directional road segment of Chengxian Road, Nanjing City, China. As illustrated in Fig. 4, we used cameras to record the traffic data including the density, velocity, volume and the proportion of large vehicles, cars, and bicycles every 15 s. The width of each lane is 3 m ($N = 3 \times 2 = 6$ m), and L_{AB} = 15.5 m.

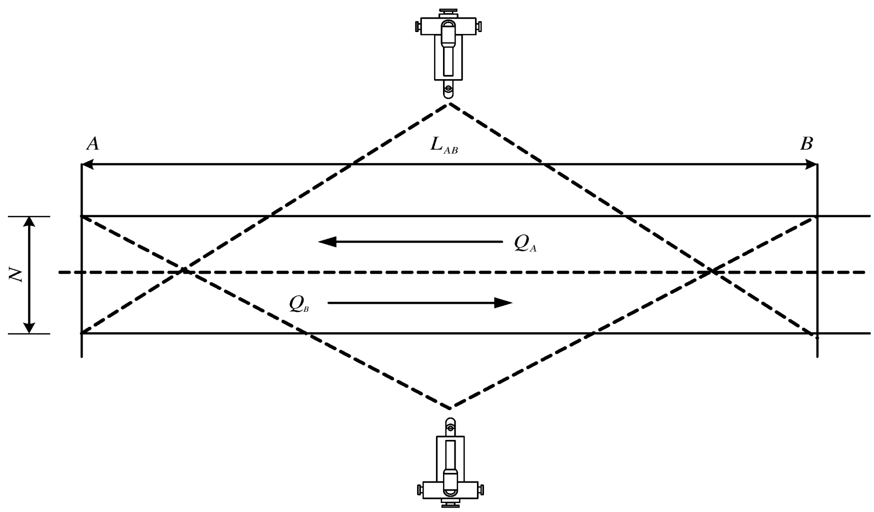

Fig. 4 Illustration of data collection method

4 Numerical study

4.1 Model installment

In simulation, a system of 2500 cells is considered under the periodic boundary condition. According to Section 2, the size of each cell is set to be 2.75 m by 3.00 m, so the system is equivalent to a road of 6.9 km long. When we start to perform a numerical simulation, the types of vehicle with a given density are initially distributed randomly on the road. After a transition time period $t_0 = 1000$ time steps (each time step equals 1 s in the real world), we start to record the time-averaged velocity of traffic flow and the total volume in every period of T (1000 time steps). Finally, we obtain the average velocity and total volume in a run, i. e. the density of total traffic flow is

$$\rho = \sum_i \rho_i$$

The density for each type vehicle is

$$\rho_i = CN_i\, size_i / 2L$$

The total average velocity is

$$\tilde{V} = \sum_i \sum_n v_{i,n}\, b / N_i$$

The total volume is

$$q = \sum_i \sum_n \rho_i\, b v_{i,n} / N_i$$

where ρ_i is density of vehicle type i; N_i and L denote the total number of vehicle in type i and length of road segment, respectively; $size_i$ is the size of vehicle type i (for bicycle, $size_i$ is $1 \times 1/3 = 1/3$, for big vehicle, $4 \times 1 = 4$, and for car, $2 \times 2/3 = 4/3$); the parameter C is used to convert the non-dimensional density into a dimensional form and equals to 363.4 ($C = 1000/2.75$); b is used to change the velocity measured by cells into meters; q is total volume with the consideration of car equivalence principle for mixed traffic flow in China (Ren et al. 2003).

Together with the observation and calibration process using the field data, the key parameters, including maximal velocity, randomization probabilities and lane changing probability for each type of vehicle, are given in Tab. 1. Accordingly, the parameter b used to change the velocity measured by cells into meters is set to be 4. In Tab. 2, a volume comparison between the field data and simulation result is given indicating an average error of 4.7%, where the M/S represents the ratio of big vehicle to car.

4.2 Numerical analysis

In order to capture the consecutive change in operational condition of mixed traffic flow due to the variation of vehicle composition, we increase the proportion of bicycles based on a given ratio of the big vehicles (buses/trucks) to cars, and then smoothly add more vehicles until all cells are occupied.

Tab. 1 Essential parameters

Vehicle type	$v_{i,\max}$	p_i	p_i^{opp}	p_i'	gap_i^{B}	gap_i^{opp}
Car	12 m/s (4 cells)	0.18	0.20	0.26	6 m (3 cells)	21m (7 cells)
Bicycle	6 m/s (2 cells)	0.13	0.40	0.20	3 m (1 cell)	12 m (4 cells)
Big vehicle	9 m/s (3 cells)	0.18	0.25	0.26	6 m (2 cells)	27 m (9 cells)

Tab. 2 Volume comparison between simulation result and field data

Bicycle ratio	*M/S* ratio	Density	Simulation result(vph)	Field data(vph)	Error(%)
0	3:1	0.40	2722	2688	1.26
0.1	2:1	0.22	1375	1396	1.50
0.2	1:1	0.23	952	912	4.39
0.4	1:2	0.31	1411	1310	7.71
0.6	1:3	0.33	1533	1478	3.72
0	3:1	0.35	2736	2789	1.90
0.1	2:1	0.19	1433	1502	4.59
0.2	1:1	0.45	1750	1688	3.67
0.4	1:2	0.32	1928	1875	2.83
0.6	1:3	0.26	1514	1526	0.79
0	3:1	0.28	1922	1847	4.06
0.1	2:1	0.12	1222	1310	6.72
0.2	1:1	0.22	987	957	3.13
0.4	1:2	0.32	1497	1526	1.90
0.6	1:3	0.40	1601	1497	6.95
0	3:1	0.41	2854	2777	2.77
0.1	2:1	0.32	1319	1260	4.68
0.2	1:1	0.34	1772	1712	3.50
0.4	1:2	0.22	1294	1287	0.54
0.6	1:3	0.18	1475	1447	1.94
0	3:1	0.20	1314	1322	0.61
0.1	2:1	0.33	1369	1392	1.65
0.2	1:1	0.17	956	914	4.60
0.4	1:2	0.22	1272	1181	7.71
0.6	1:3	0.15	1198	1101	8.81

4.2.1 Volume-density analysis

Figures 5-7 illustrate the volume-density analysis results: 1) The maximal volume increases firstly and decreases latterly with the increase of density. 2) With the increase in the proportion of the big vehicle the total volume decreases. As big vehicles require larger spaces for operations while associated with a relatively low speed, all vehicles therefore have a lower probability to conduct overtakes but follow their front ones with the consideration of safety. 3) Bicycles have a great impact to the traffic condition in terms of the total volume. The total volume drops dramatically by 30% after we add 10% bicycle into the flow which is only consisted by autos (big vehicles and cars). As bicycles have more flexibility with smaller sizes and higher probabilities in lane changing when the safety criteria are met, other vehicles therefore are easily to be inferred and forced to follow them with a relatively low speed. 4) The maximal total volume starts to recover when the proportion of bicycle is higher than 10%. Though the bicycles have a low velocity, their advantages lying on the low requirement for the space can compensate such deficiency and increase the traffic volume by adding more bicycles into the network. 5) The phenomenon of double-summit in terms of the total volume appears when the proportion of bicycle is beyond 60%. One reasonable explanation is that: for the first summit, with the increase of the density (lower than 0.2), vehicles with higher velocity may change their lanes to avoid the interference caused by their front vehicles with a relative low speed. While for the consideration of safety requirement, the total volume drops along with decrease in opportunities of lane changing when the density is higher than 0.2. However, when the density approximately reaches 0.45, the whole traffic may operate with a low speed and bicycles therefore can find more chances to change their lanes and eventually lead to the increase in total volume.

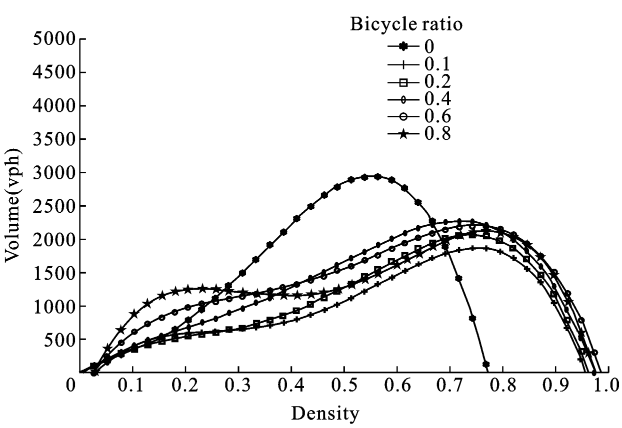

Fig. 5 Volume-density curves when the ratio of big vehicles to cars is 3:1

4.2.2 Velocity-density analysis

As shown in Figs. 8-10, we illustrate the relation between velocity and density with different compositions of vehicle types. The following findings can be reached: 1) Bicycles have a great impact on the average velocity. Compared with the velocity of traffic flow only consisted by big vehicles and cars, the operational speed drop 60% approximately when the bicycle is added. 2) The variation of velocity changes in a small range when the proportion of bicycle is higher than 10%. Because most autos (big vehicles and cars) are following their front vehicles with a slow speed after the incorporation of bicycles, the space requirement for bicycles moving with their maximal speed is relatively easy to be satisfied. The average speed of the flow is therefore concentrated around the maximal velocity of bicycles. 3) As the cars have higher maximal velocities, the increase in the proportion of cars will lead to the increase in total average speed though it is not significant when the bicycles are taken into account.

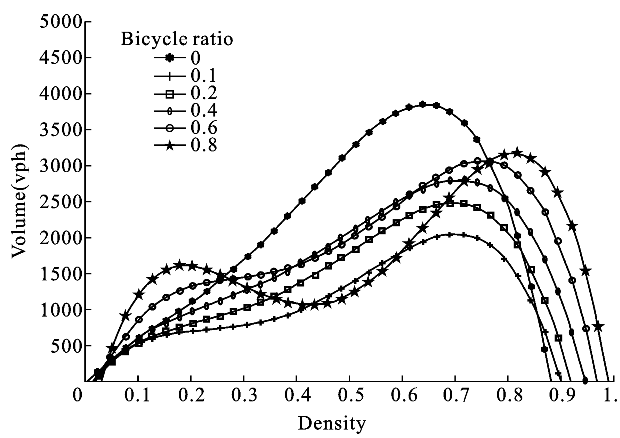

Fig. 6 Volume-density curves when the ratio of big vehicles to cars is 1:1

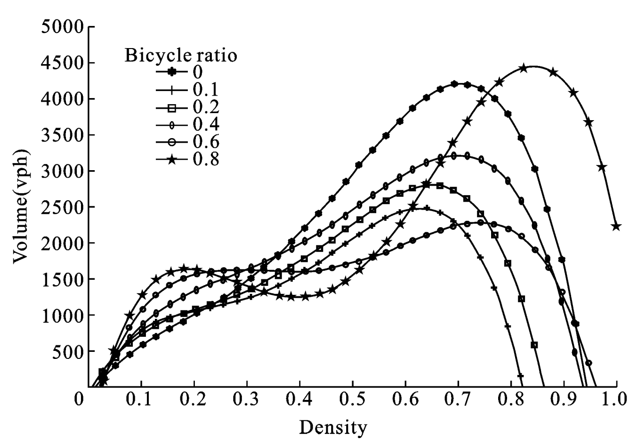

Fig. 7 Volume-density curves when the ratio of big vehicles to cars is 1:3

5 Conclusions

In summary, in this paper, we build a CA model to investigate the operational characteristics of mixed traffic flow at a two lane bi-directional road segment. To

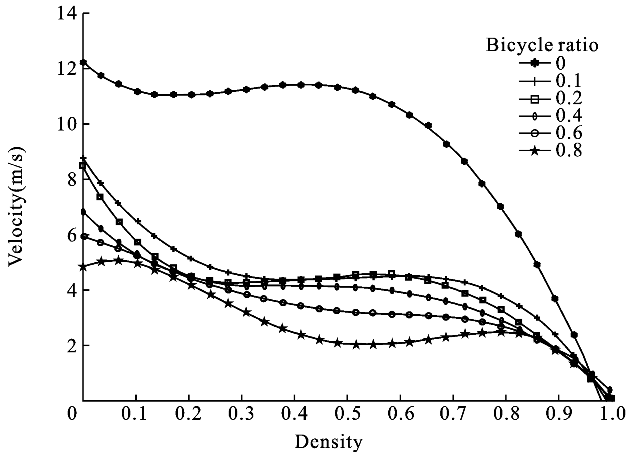

Fig. 8 Speed-density curves when the ratio of big vehicles to cars is 3:1

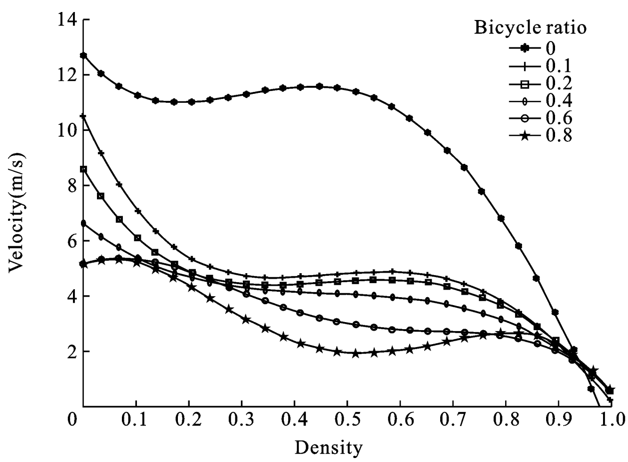

Fig. 9 Speed-density curves when the ratio of big vehicles to cars is 1:1

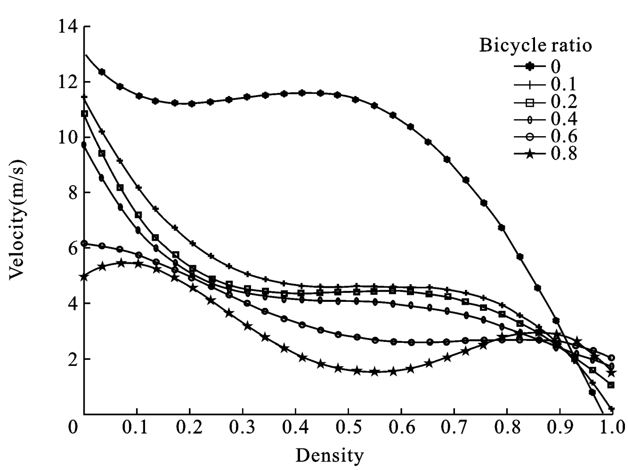

Fig. 10 Speed-density curves when the ratio of big vehicles to cars is 1:3

capture more reality, we categorize vehicles into different groups including big vehicles (buses/trucks), cars, and bicycles, and take into account their physical and mechanic differences. Essential parameters with regards to different vehicle types are calibrated by using the field data collected at a two-lane bi-directional road segment in Nanjing City, China. We have detailed the relations of volume-density and velocity-density under the various compositions of traffic flow which leads to some key findings including: 1) Bicycles impact the traffic conditions in terms of volume and velocity to a large extent that is the average speed and maximal volume drop 60% and 30% respectively after the incorporation of 10% bicycles into the mixed traffic flow. 2) The volume starts to recover when the proportion of bicycles is higher than 10%. 3) The phenomenon of double summits of the traffic volume appears when the proportion of bicycles is higher than 60%. 4) The average speed of total traffic flow is highly concentrated after the incorporation of bicycles.

In the next step, this model will be extended to incorporate signals at intersection, and queuing behavior and stop-and-go conditions will be explicitly modeled. Behaviors and impact of pedestrian's will be taken into account to describe more complex traffic flow at both intersections and road segments. Further, the situation of multiple lanes (more than 2) will be considered as which may induce other lane changing behaviors having a great influence on the traffic conditions (i. e. change to another in the same direction).

References

Adams, W. F., 1936. Road traffic considered as a random series. Journal of the ICE, 4(1): 121-130.

Basso, L. J., Guevara, C. A., Gschwender, A., et al., 2011. Congestion pricing, transit subsidies and dedicated bus lanes: efficient and practical solutions to congestion. Transport Policy, 18(5): 676-684.

Chander, R. E., Herman, R., Montroll, E. W., 1958. Traffic dynamics: studies in car following. Operations Research, 6(2): 165-184.

Cremer, M., Ludwig, J., 1986. A fast simulation model for traffic flow on the basis of boolean operations. Mathematics and Computers in Simulation, 28(4): 297-303.

Foulaadvand, M., Belbasi, S., 2007. Vehicular traffic flow at a non-signalized intersection. Journal of Physics A: Mathematical and Theoretical, 40(29): 8289-8297.

Fukui, M., Ishibashi, Y., 1996. Traffic flow in 1D cellular automaton model including cars moving with high speed. Journal of the Physical Society of Japan, 65(6): 1868-1870.

Greenshields, B. D., Shapiro, D., Erickson, E. L., 1947. Traffic performance at urban street intersections. Bureau of Highway Traffic, Yale University, New Haven.

Gundaliya, P., Mathew, T., Dhingra, S., 2008. Heterogeneous traffic flow modelling for an arterial using grid based approach. Journal of Advanced Transportation, 42(4): 467-491.

Herman, R., Montroll, E. W., Potts, R. B., et al., 1959. Traffic dynamics: analysis of stability in car following. Operations Research, 7(1): 86-106.

Jiang, R., Jia, B., Wu, Q., 2004. Stochastic multi-value cellular automata models for bicycle flow. Journal of Physics A: Mathematical and General, 37(6): 2063-2072.

Khan, S. I., Maini, P., 1999. Modelling heterogeneous traffic flow. Transportation Research Record, 1678: 234-241.

Kinzer, J. P., 1993. Application of the theory of probability to problems of highway traffic. Traffic Engineering and Control, 5: 284-287.

Lighthill, M. J., Whitham, G. B., 1955. On kinematic waves Ⅱ: a theory of traffic flow on long crowded roads. Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, 229(1178): 317-345.

Mallikarjuna, C., Rao, K., 2009. Cellular automata model for heterogeneous traffic. Journal of Advanced Transportation, 43(3): 321-345.

Meng, J. P., Dai, S. Q., Dong, L. Y., et al., 2007. Cellular automaton model for mixed traffic flow with motorcycles. Physica A: Statistical Mechanics and its Applications, 380(1/2): 470-480.

Nagel, K., Schreckenberg, M., 1992. A cellular automaton model for freeway traffic. Journal de Physique Archives, 2(12): 2221-2229.

Nishinari, K., Takahashi, D., 1998. Analytical properties of ultradiscrete Burgers equation and rule-184 cellular automaton. Journal of Physics A: Mathematical and General, 31(24): 5439-5450.

Payne, H. J., 1971. Models of freeway traffic and control. Mathematical Models of Public Systems, 1(1): 51-61.

Payne, H. J., 1979. FREFLO: a macroscopic simulation model of freeway traffic. Transportation Research Record, 722: 68-77.

Ren, F. T., Liu, X. M., Rong, J., 2003. Traffic engineering science. China Communications Press, Beijing.

Ruskin, H. J., Wang, R., 2007. Lecture notes in computer science, 2329: 381.

Vasic, J., Ruskin, H. J., 2012. Cellular automata simulation of traffic including cars and bicycles. Physica A: Statistical Mechanics and its Applications, 391(8): 2720-2729.

Xie, D. F., Gao, Z. Y., Zhao, X. M., et al., 2009. Characteristics of mixed traffic flow with non-motorized vehicles and motorized vehicles at an unsignalized intersection. Physica A: Statistical Mechanics and its Applications, 388(10): 2041-2050.

Zhao, X. M., Gao, Z. Y., 2005. Controlling traffic jams by a feedback signal. The European Physical Journal B: Condensed Matter and Complex Systems, 43(4): 565-572.