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Predicting shear strength of RC interior beam–column joints by modified rotating-angle softened-truss model

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ABSTRACT

A theoretical model is presented for analysing the shear behaviour and predicting the shear strength of reinforced concrete (RC) interior beam–column joints. The model presented is referred to as the modified rotating-angle softened-truss model (MRA-STM), which is modified from the rotating-angle softened-truss model and the modified compression field theory. In the proposed methodology, the RC interior joint is treated as an RC shear panel that is subjected to vertical and horizontal shear stresses transferred from adjacent columns and beams. Employing the deep beam analogy, the characteristic strut and truss actions typical in beam–column joints are represented by the effective transverse compression stresses and the softened concrete truss in the model. Sixteen RC interior beam–column joints were subsequently analysed with the proposed model. Shear strengths of the RC interior beam–column joints predicted by the proposed model show very good agreement with the experimental results.

1. Introduction

Beam–column joints play an important role in transferring bending moments and shear forces between adjacent beams and columns in a moment-resisting frame. When the frame is subjected to applied lateral loads, bending moments of beams and columns across the attached joint will induce very large horizontal and vertical shear forces in the joint, which may be typically many times larger than those in adjacent beams and columns. It has been widely recognised that properly detailed beam–column joints are essential in maintaining the integrity and stability of ductile frame structures under seismic actions.

Although a large amount of research into RC beam–column joints has been conducted during the past three decades, the resisting mechanisms of shear forces in a beam–column joint core are still in dispute. Uncertainties regarding to the behaviour of RC beam–column joints are attributed to the numerous structural parameters involved and interaction of different internal forces in the joints. Hence, currently there is no general consensus of opinion on how to reasonably and accurately analyse and design RC beam–column joints.

During the recent years, great progress has been made in the theoretical development of predicting the ultimate shear strength and shear deformation of reinforced concrete membrane elements throughout their loading histories. These proposed theories are mainly based on the truss model concept, where the cracked concrete is treated as a continuous material, so that the Navier's principles of mechanics of materials, i.e. the stress equilibrium, Mohr compatibility conditions and constitutive laws of materials of cracked concrete and reinforcement, can be satisfied. The typical and rational models, including the modified compression field theory (MCFT) [1], the rotating angle softened truss model (RA-STM) [2] and the fixed angle softened truss model (FA-STM) [3], have been developed for predicting the nonlinear shear behaviour of cracked reinforced concrete membrane elements. Some of these theories have been shown to be used for the analysis of low-rise shear walls, framed shear walls and deep beams [4]. Recently, a modified rotating-angle softened-truss model (MRA-STM) has been proposed for the analysis of reinforced concrete membranes in shear and the strength prediction of reinforced concrete exterior beam–column joints [5].

In this paper, the modified rotating-angle softened-truss model is presented for analysing the shear behaviour of RC interior beam–column joints, where the joints are treated as RC shear panels subjected to vertical and horizontal shear stresses transferred from adjacent columns and beams. Employing the deep beam analogy, the characteristic truss action typical in beam–column joints is represented by the effective transverse compression stresses, which can be effectively calculated. The MRA-STM is shown to provide an effective, yet accurate, means of predicting the shear strength of RC interior beam–column joints.

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2. Modified rotating-angle softened-truss model

The modified rotating-angle softened-truss model (MRA-STM) for predicting the shear behaviour and strength of reinforced concrete membrane elements is derived based on the concepts of RA-STM and MCFT, where "the concrete contribution" in cracked concrete membrane elements has been adequately considered in the proposed model. The detailed development of the model is described [4] and summarised as follows.

2.1. Formulation of stresses and strains

The stresses of reinforced concrete membrane elements subjected to shear are shown in Fig. 1. It is assumed that the angle of cracks in the concrete, α, is non-coincident with the angle of the concrete principal compressive stress, θ, and is kept rotating in correspondence with the level of stresses.

Fig. 2 shows the crack and principal angles in Mohr circles of the average stress and average strain for cracked concrete. From the Mohr's circle of the average stress, the corresponding average stresses of cracked concrete in the proposed model are expressed by

$$f_{cx} = f_{c1} - v_{cxy} \cot\theta \tag{1}$$

$$f_{cy} = f_{c1} - v_{cxy} \tan\theta \tag{2}$$

$$v_{cxy} = \frac{f_{c1} - f_{c2}}{2} \sin 2\theta \tag{3}$$

where f_{cx} and f_{cy} are the average concrete stresses in x and y directions, respectively; f_{c1} and f_{c2} are the average principal stresses of concrete in 1 and 2 directions, respectively; v_{cxy} is the average shear stress of concrete in the x-y coordinate.

Similarly, from the Mohr's circle of the average strain, the corresponding average strains of cracked concrete in the proposed modified rotating-angle softened-truss model are given by

$$\varepsilon_{cx} = \frac{\varepsilon_{c1} - \varepsilon_{c2}}{2}(1 - \cos 2\theta) + \varepsilon_{c2} \tag{4}$$

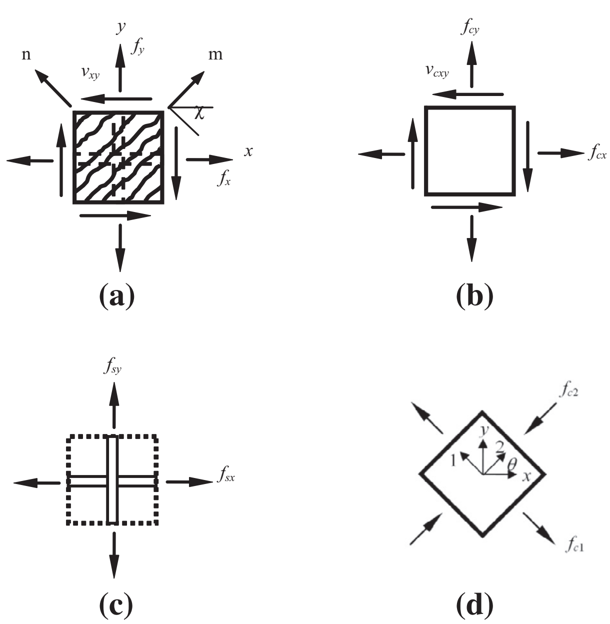

Fig. 1. Stresses of cracked reinforced concrete membrane elements subjected to shear. (a) Cracked reinforced concrete membrane; (b) concrete membrane; (c) steel reinforcement; (d) principal angle and stresses.

$$\varepsilon_{cy} = \varepsilon_{cx} + (\varepsilon_{c1} - \varepsilon_{c2}) \cos 2\theta \tag{5}$$

$$\gamma_{cxy} = 2(\varepsilon_{cy} - \varepsilon_{c2}) \tan\theta \tag{6}$$

where ε_{cx} and ε_{cy} are the average concrete strains in x- and y-directions, respectively; ε_{c1} and ε_{c2} are the average principal strains of concrete in 1 and 2 directions, respectively; γ_{cxy} is the average shear strain of concrete in the x-y coordinate.

2.2. Constitutive laws for material

In deriving the MRA-STM, the average compressive stress–strain relationships of concrete in compression proposed by Belarbi and Hsu [6] are adopted,

$$f_{c2} = \zeta f'_c \left[2\left(\frac{\varepsilon_{c2}}{\zeta\varepsilon_o}\right) - \left(\frac{\varepsilon_{c2}}{\zeta\varepsilon_o}\right)^2 \right] \quad (\varepsilon_{c2}/\zeta\varepsilon_o \le 1) \tag{7}$$

$$f_{c2} = \zeta f'_c \left\{ 1 - \left[\left(\frac{\varepsilon_{c2}}{\zeta\varepsilon_o} - 1\right) \Big/ \left(\frac{4}{\zeta} - 1\right) \right]^2 \right\} \quad (\varepsilon_{c2}/\zeta\varepsilon_o > 1) \tag{8}$$

where f'_c is the cylinder strength of concrete, ε_o is the concrete strain at peak stress, ε_{c2} is the average principal compressive strain of concrete, and ζ is the softening coefficient, given by

$$\zeta = \frac{5.8}{\sqrt{f'_c}} \frac{1}{\sqrt{1 + 400\varepsilon_{c1}}} \leqslant \frac{0.9}{\sqrt{1 + 400\varepsilon_{c1}}} \tag{9}$$

in which ε_{c1} is the average principal tensile strain of concrete.

The average tensile stress–strain relationship of concrete is given as follows,

$$f_{c1} = E_c \varepsilon_{c1} \quad (\varepsilon_{c1} \leqslant \varepsilon_{cr}) \tag{10}$$

$$f_{c1} = f_{cr} \left(\frac{0.00008}{\varepsilon_{c1}}\right)^{0.4} \quad (\varepsilon_{c1} > \varepsilon_{cr}) \tag{11}$$

where $E_c = 3875\sqrt{f'_c}$ (MPa) is Young's modulus of concrete, $\varepsilon_{cr} = 0.00008$ is the cracking strain of concrete, and $f_{cr} = 0.31\sqrt{f'_c}$ (MPa) is the cracking strength of concrete.

The following bilinear stress–strain relationships are adopted for steel reinforcement,

$$f_s = E_s \varepsilon_s \quad (\varepsilon_s \leqslant \varepsilon_y) \tag{12}$$

$$f_s = f_y \quad (\varepsilon_s > \varepsilon_y) \tag{13}$$

where E_s (N/mm^2) is Young's modulus of steel reinforcement; f_s is the steel stress along x or y axis, expressed by f_{sx} and f_{sy} (N/mm^2); f_y (N/mm^2) is the steel strength along x or y axis; ε_s is the strain of reinforcement along x or y axis.

3. Analysis of interior beam–column joints by MRA-STM

3.1. Deep beam analogy

Consider a concrete deep beam with simple supports subjected to two concentrated loads, as shown in Fig. 3. The arch action between the applied loads and the support reactions can be modelled by introducing the effective transverse compressive stress p [7]. The transverse stress p is directly proportional to the shear stress v and the shear span-to-depth ratio, given by

$$p = Kv \tag{15}$$

where

$$K = \frac{2d_v}{h} \quad (0 < a_v/h \leqslant 0.5) \tag{15a}$$

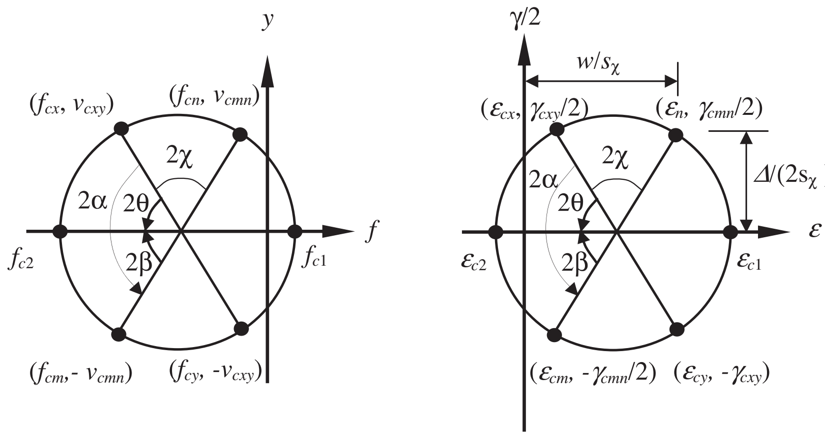

Fig. 2. Mohr circles of average stress and average strain for cracked concrete.

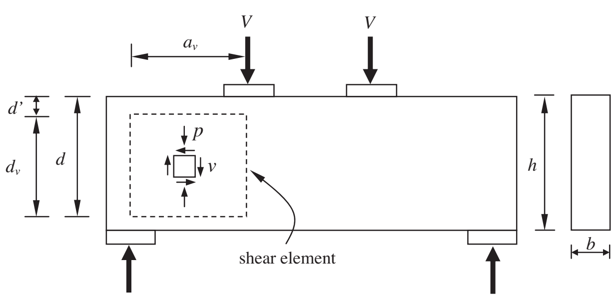

Fig. 3. Concrete stresses within shear span of a deep beam.

$$K = \frac{4}{3}\frac{d_v}{a_v}\left(1 - \frac{a_v}{2h}\right) \quad (0.5 < a_v/h \leqslant 2) \tag{15b}$$

$$K = 0 \quad (a_v/h > 2) \tag{15c}$$

Similar to the shear element of a deep beam shown in Fig. 4, the shear element in the joint core of an RC interior beam–column joint is under the action of horizontal and vertical shears induced by the attached beams and columns. Two effective transverse compressive stresses in both horizontal and vertical directions, p_1 and p_2, are introduced. In general, the resistance of shear forces in an RC beam–column joint are based on the two postulated mechanisms: strut mechanism and truss mechanism [8]. The truss mechanism consists of the contribution to the shear resistance of the horizontal and vertical reinforcement inside the joint core. This shear resistance is mainly influenced by the shear strength of the cracked concrete. The strut mechanism transfers shear forces through a diagonal concrete strut that sustains compression only and is assumed to be inclined at an angle close to that of the potential corner-to-corner failure plane of the joint. This shear resistance is significantly influenced by the span-to-depth ratio of the joint, which is similar to the arch action of the deep beam shown in Fig. 3. Similar to the strut mechanism, the effective transverse stresses p_1 and p_2 and shear stress v shown Fig. 4 can be directly related by a factor that is dependent upon the shear span-to-depth ratios of the attached beams and columns.

It is considered that the effective shear element in an interior joint core is bounded by the main steel reinforcement of the adjacent columns and beams, as shown in Fig. 5; where d_{sh} and d_{sv} are the horizontal and vertical dimensions of the shear element, l_h and l_v are the horizontal and vertical spans of the joint, and h_b and h_c are the beam and column depths, respectively. The effective transverse stress, p_1, of the joint shown in Fig. 4 is analogous to the effective transverse stress, p, of the deep beam shown in Fig. 3, which is calculated by Eq. (14). Hence, the effective transverse stress, p_1, of the joint shown in Fig. 5 is given by

$$p_1 = K_1 v \tag{16}$$

where

$$K_1 = \frac{2d_{sh}}{h_c} \quad (l_v/h_c \leqslant 0.5) \tag{17a}$$

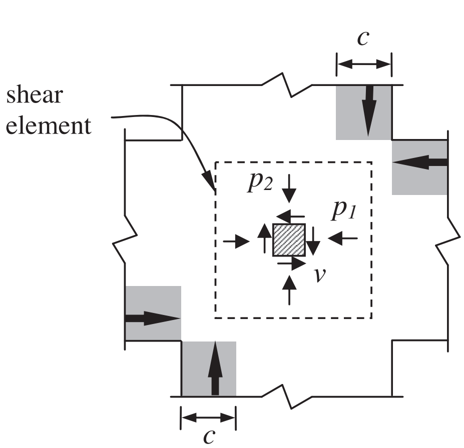

Fig. 4. Stresses in an interior beam-column joint.

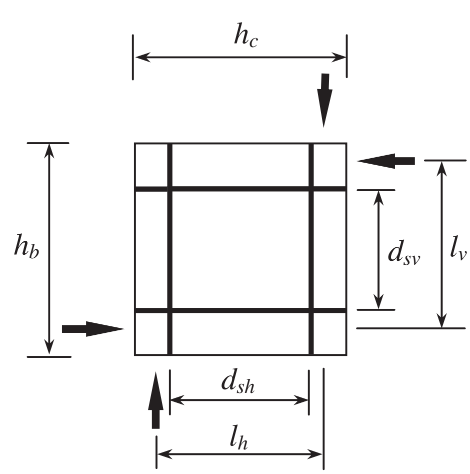

Fig. 5. Dimensions of an interior beam–column joint.

$$K_1 = \frac{4}{3}\frac{d_{sh}}{l_v}\left(1 - \frac{l_v}{2h_c}\right) \quad (0.5 < l_v/h_c \leqslant 2) \tag{17b}$$

Similarly, the effective transverse stress of the joint, p_2, is given by

$$p_2 = K_2 v \tag{18}$$

where

$$K_2 = \frac{2d_{sv}}{h_b} \quad (l_h/h_b \leqslant 0.5) \tag{19a}$$

$$K_2 = \frac{4}{3}\frac{d_{sv}}{l_h}\left(1 - \frac{l_h}{2h_b}\right) \quad (0.5 < l_h/h_b \leqslant 2.0) \tag{19b}$$

In Fig. 4, the depth of the flexural compression zone, c, of the columns connected to the joint can be estimated [8] by

$$c = \left(0.25 + 0.85\frac{N}{f_c' A_g}\right)h_c$$

where N is the axial compressive load on column (N), f'_c is the compressive strength of standard concrete cylinder (N/mm^2), A_g is the gross area of column section (mm^2), and h_c is the column depth (mm).

Hence, the vertical span of the joint l_h shown in Fig. 5 is equal to $(h_c - c)$. For beam sections, the moment arm l_v of the horizontal forces shown in Fig. 5 can be taken as $0.9d$ [9], where d is the effective depth of the beam. The effective transverse stresses in the joint, p_1 and p_2, can then be determined using Eqs. (16) and (18), respectively.

3.2. Solution scheme

In the proposed MRA-STM, to determine the steel stresses and check the stress equilibrium of reinforced concrete membrane elements, the total strains should be determined first. Fig. 6 shows the strain state of a concrete strut, where ε_x and ε_y are the total average smeared-strains of the cracked concrete in x and y directions, respectively.

Before obtaining the total average strains of reinforced concrete membrane elements, local strains at cracks are calculated, where the kinematic conditions at the crack interface should be considered due to the crack opening and slip deformation. The crack width w and slip displacement Δ are directly proportional to the crack spacing s_χ, the normal strain ε_{cn} and shear strain γ_{cmn} across cracks, given by

$$w = s_\chi \varepsilon_{cn} \tag{20}$$

$$\Delta = s_\chi \gamma_{cmn} \tag{21}$$

$$\varepsilon_{cn} = \varepsilon_{c2}\sin^2\beta + \varepsilon_{c1}\cos^2\beta \tag{22}$$

$$\gamma_{cmn} = (\varepsilon_{c1} - \varepsilon_{c2})\sin 2\beta \tag{23}$$

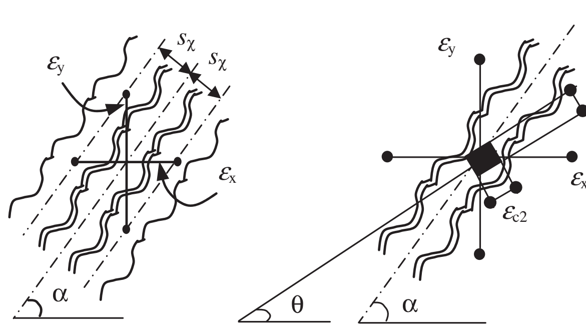

Fig. 6. Strain state of concrete strut.

$$s_\chi = 1/(\cos\chi/s_{mx} + \sin\chi/s_{my}) \tag{24}$$

where α is crack angle, β is the angle between α and θ equal to $(\alpha - \theta)$ and s_{mx} and s_{my} are the mean values of crack spacing in x- and y-directions, respectively.

The longitudinal, transverse and shear strains caused by crack opening, ε_{wx}, ε_{wy} and γ_{wxy}, and the longitudinal, transverse and shear strains caused by crack slipping, $\varepsilon_{\Delta x}$, $\varepsilon_{\Delta y}$ and $\gamma_{\Delta xy}$, are calculated by

$$\varepsilon_{wx} = \frac{w}{s_\chi}\cos^2\chi, \quad \varepsilon_{wy} = \frac{w}{s_\chi}\sin^2\chi, \quad \gamma_{wxy} = \frac{w}{s_\chi}\sin 2\chi \tag{25}$$

$$\varepsilon_{\Delta x} = -\frac{\Delta}{2S_\chi}\sin 2\chi, \quad \varepsilon_{\Delta y} = \frac{\Delta}{2S_\chi}\sin 2\chi, \quad \gamma_{\Delta xy} = \frac{\Delta}{S_\chi}\cos 2\chi \tag{26}$$

where χ is the crack orientation, which is equal to $(90^\circ - \alpha)$.

The crack spacing s_χ may be calculated by following the procedure proposed by Vecchio and Collins [1]. By combining the average strains determined by Eqs. (20)–(23) and the local strains caused by crack opening and slipping calculated by Eqs. (25) and (26), the total strains of the concrete membrane elements are given by

$$\varepsilon_x = \varepsilon_{cx} + \varepsilon_{wx} + \varepsilon_{\Delta x} \tag{27}$$

$$\varepsilon_y = \varepsilon_{cy} + \varepsilon_{wy} + \varepsilon_{\Delta y} \tag{28}$$

$$\gamma_{xy} = \gamma_{cxy} + \gamma_{wxy} + \gamma_{\Delta xy} \tag{29}$$

The total applied stresses and shear capacity can be determined by

$$f_x = f_{cx} + \rho_{sx} f_{sx} = -p_1 \tag{30}$$

$$f_y = f_{cy} + \rho_{sy} f_{sy} = -p_2 \tag{31}$$

$$v_{xy} = v_{cxy} \tag{32}$$

where ρ_{sx} and ρ_{sy} are the steel reinforcement ratios in x and y directions, respectively; f_{sx}, f_{sy} are the steel reinforcement stresses in x and y directions, respectively. It is clear that if a reinforced concrete membrane element is subjected to shear as well as transverse compressive stresses in x and y directions, the total applied stresses $f_x = -p_1$ and $f_y = -p_2$.

When a reinforced concrete membrane element is under the action of increasing shear forces, the first set of cracks will form in the major principal concrete stress direction if the principal concrete stress reaches the tensile strength of the concrete. The orientation of the crack angle remains constant until the principal concrete stress exceeds the tensile strength. New cracks will then form in the major principal stress direction. Hence, it is assumed that the set of currently open cracks will close and disappear. The crack angle of the concrete changes from the initial crack to the point of failure.

3.3. Solution procedure

A flow chart of the solution procedure for predicting the shear strength of RC interior beam–column joints by MRA-STM is presented in Fig. 7. The proposed model can capture the effects of vertical and transverse reinforcement and the joint span-to-depth ratio on the shear behaviour and shear strength of the beam–column joints, where the effect of the joint span-to-depth ratio is reflected by the effective transverse compression stresses p_1 and p_2.

It has been shown that the smaller the span-to-depth ratio of a joint, the larger the effective transverse compressive stresses, thus resulting in the higher shear strength of the joint. On the other

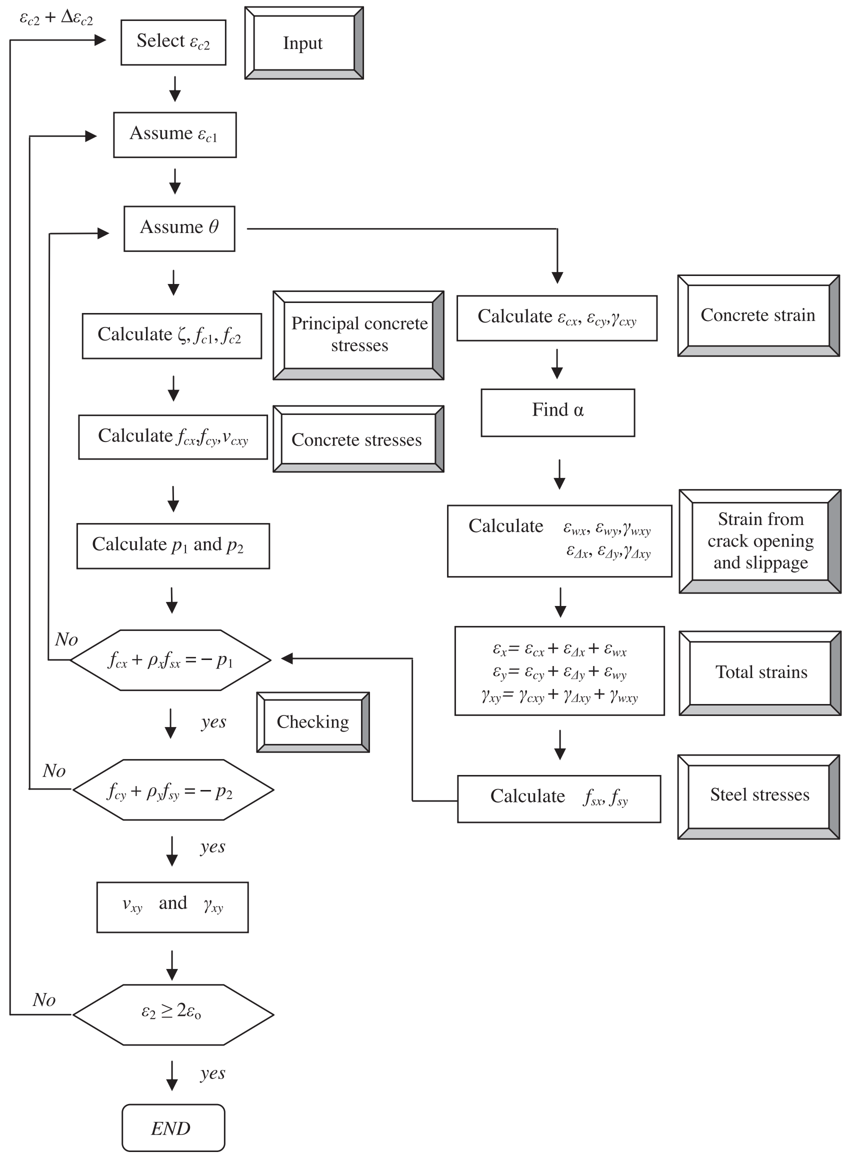

Fig. 7. Flow chart of computational solution procedure for determining shear stress–strain relationship of RC interior beam–column joints by MRA-STM.

hand, the axial load level on columns is also considered as one of the main factors affecting the shear strength of joints, whereas the axial load has no further effect on the joint shear strength if the ratio l_h/h_b is smaller than 0.5. This may explain why a high axial load does not exhibit the beneficial effect on the shear strength of RC beam–column joints [10].

4. Experimental verification

Sixteen RC interior beam–column joints are analysed using the proposed modified rotating-angle softened-truss model. Among these joint specimens, five specimens were tested by Pessiki et al. [11], four specimens were tested by Fuji et al. [12] and seven specimens were tested by Meinheit et al. [13]. Results of experimental shear strength of the beam–column joints, V_{exp}, and comparisons with the theoretical predictions, V_{pred}, are presented in Table 1.

Table 1
Experimental and predicted shear strengths of interior beam–column joints.

Tests	Specimen	f_c' (MPa)	V_{exp} (kN)	V_{pred} (kN)	V_{exp}/V_{pred}
Pessiki et al.	1	32.7	984.9	939.0	1.05
	2	32.5	968.2	939.0	1.03
	3	30.4	935.8	939.0	1.00
	4	31.9	922.6	961.8	0.96
	5	29.8	954.3	940.2	1.02
Fujii & Morita	A1	40.2	412	411.8	1.00
	A2	40.2	380	411.8	0.92
	A3	40.2	412	422.8	0.97
	A4	40.2	420	434.7	0.97
Meinheit & Jirsa	I	26.2	1089.4	1244.2	0.88
	II	41.8	1596.3	1553.9	1.02
	III	26.6	1227.2	1275.1	0.96
	V	35.9	1529.6	1412.8	1.08
	VI	36.8	1645.2	1535.1	1.07
	XII	35.2	1947.5	1458.0	1.34
	XIII	41.3	1556.2	1560.5	1.00

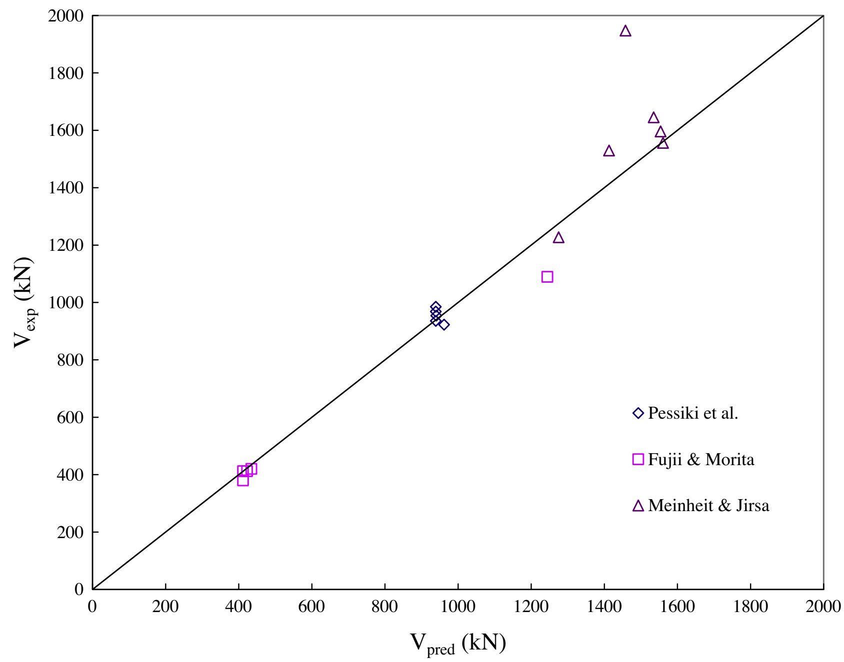

Fig. 8. Correlation of experimental and predicted joint shear strengths.

From Table 1, the overall mean ratio of the experimental shear strengths of interior beam–column joints to the theoretical predictions, V_{exp}/V_{pred}, is 1.01. The corresponding coefficient of variance is 10%. The correlation of the experimental and predicted joint shear strengths is shown in Fig. 8. Hence, two sets of the results show very good agreement.

5. Conclusion

In this paper, a theoretical model is presented for analysing the shear behaviour and predicting the shear strength of reinforced concrete interior beam–column joints. The model presented is referred to as the modified rotating-angle softened-truss model (MRA-STM), which is modified from the rotating-angle softened-truss model and the modified compression field theory. In the proposed methodology, the RC interior joint is treated as an RC shear panel that is subjected to vertical and horizontal shear stresses transferred from the attached columns and beams. By employing the deep beam analogy, the characteristic strut and truss actions typical in RC beam–column joints are represented by the effective transverse compression stresses and the softened concrete truss in the model. Sixteen RC interior beam–column joints with various joint steel ratios, beam and column depths and axial load ratios were subsequently analysed with the proposed model. Shear strengths of the RC interior beam–column joints predicted by the proposed model show very good agreement with the experimental results. The proposed MRA-STM combined with the deep beam analogy is shown to provide an effective, yet accurate, analytical means of predicting the shear strength of RC interior beam–column joints.

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