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A branch and bound method for solving multi-factory supply chain scheduling with batch delivery

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ABSTRACT

This study addresses the scheduling of supply chain with interrelated factories containing suppliers and manufacturers. These elements of the chain are positioned in series and thus the efficient design of the link among them would insure good performance of the whole. In this paper, jobs transportation among factories and also delivery to the customer can be performed by batch of jobs. The capacity of each batch is limited and the cost per batch delivery is fixed and independent of the number of jobs in the batch. Thus decision should be made on the number of batches, assignment of each job to a batch and also production and delivery scheduling of batches in each factory. The problem scrutinization is on the tradeoff between minimizing transportation cost and tardiness cost. A branch and bound method for solving this problem is presented. A lower bound and a standalone heuristic which is used as an upper bound are also introduced. Computational tests are conducted to evaluate the performance of the proposed method.
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1. Introduction

Globalization of nowadays markets and therefore increasing competitiveness, invoke manufacturer to consider activities from supplier to customer instead of their own plant's activities. In the other words, they tend to design and optimize the whole supply chain. Supply chain is an improved organization which contains suppliers, customers, a number of companies, products and services. The integration and coordination of these components of the chain would result in a reliable flow of goods, services and information. Thus, sharing information, coordination in planning and scheduling of manufacturing facilities in the same supply chain, may leads to performance enhancement, higher reliability, lower inventory and, etc.

The significance of distributed scheduling problem in a multi-factory production network has been one of the hottest topics in recent years (Chung, Chan, & Chan, 2009). As there are number of factories along the supply chain, the scheduling activities are more complex than the traditional single-factory scheduling problems (Moon & Seo, 2005).

Different positioning of the factories along the supply chain may leads to different structure and characteristics of the system. Factories can be positioned in parallel, serial or network structure. In parallel structure, multiple factories which are considered to be able to produce various types of product are positioned in a parallel structure. Sauer (1998), Karatza (2001), Guinet (2001), Moon, Kim, and Hur (2002), Jia, Nee, Fuh, and Zhang (2003), Archimède, Charbonnaud, and Mercier (2003), Chan, Chung, and Chan (2005, 2006), Jia, Fuh, Nee, and Zhang (2007), Naderi and Ruiz (2010), De Giovanni and Pezzella (2010), Shah and Ierapetritou (2012) and Behnamian and Ghomi (2012) investigated the parallel multi-factory scheduling problem.

But if factories are positioned in the series, the system would have the serial structure. While the material enters the system, the first plant starts processing. After finishing the process on this plant the semi-finished product would transfer to the downstream plant and the production process would start there. If these two plants are positioned in two different geographical places, transportation constraints should be considered too (see Fig1).

Interrelation among factories in the serial multi-factory supply chain causes the high value of complexity. This means that the effect of material shortage in the upstream factories would be extended through the supply chain and cause delay in production in the downstream factories. On the other hand, stopping the production in the downstream factories because of inventory accumulation would cause decrease or stop in production of upstream factories. Thus, Production and transportation between upstream and downstream factories should be synchronized in order to decrease inventory cost and also to avoid risk of stock out for a factory (Simchi-levi, Kaminsky, & Simchi-levi, 2000). H'Mida and Lopez (2012) applied the constraint satisfaction approach for the

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Nomenclature

Indices

f	factory
m	factory
i	job
j	job

Parameters

F	number of factories
n	number of jobs to be processed
d_j	due date of job j
B	capacity of each vehicle (maximum number of jobs in a batch)
p_{jf}	processing time of job j in the factory f
M	big number
τ^f	the transportation time between factory f and $f+1$
η	the cost of tardiness
β	the cost of delivery

Decision variables

σ_{jh}	equal to 1 if job j is positioned in batch h, and 0 otherwise
Δ_h	equal to 1 if there is at least a job in batch h, and 0 otherwise
A_h	The arrival time of batch h to the customer
T_j	tardiness of job j

integrated production and transportation scheduling case study in the serial multi-site manufacturing environment. Huang and Yao (2013) investigated a problem which considers sequencing, lot-sizing and scheduling of several products which are being manufactured through several firms in a serial-type supply chain. They implemented a time-varying lot-sizing policy for problem formulation and also solved it by a three-phase heuristic.

Network structure is a combination of serial and parallel structure. The same as parallel structure, there are a number of identical plants in each stage and the allocation of orders (jobs) to factories should be considered too. Chung, Lau, Choy, Ho, and Tse (2010) studied the multi-factory scheduling problem. In this study factories are structured as network. They considered a capacity constraint, precedence relationship and production lines with parallel machines. They presented a modified genetic algorithm to minimize the completion time.

As mentioned above, since products of a factory would be delivered to the next factory as a raw material, the system's cost significantly depends on the transportation cost of products. Thus the coordination of production and transportation is a challenging problem for this collaborative environment. Cheng and Kahlbacher (1993) introduced an approach that allots number of jobs to several batches each of which would be delivered to downstream factory as a single shipment. This problem would be considered as a batch delivery problem. Designing a batch of products to be delivered is a reasonable approach in reducing the transportation cost. As it may increase the number of tardy jobs or the total tardiness, the goal of the problem is to determine simultaneously the optimal number of batches, the assignment of jobs to the batches and the scheduling of batch in order to minimize the problem's objective function. The simple example of this procedure is shown in Fig2. The objective of this paper is the summation of total transportation cost and total tardiness cost. The total transportation cost is an increasing function of the number of batches.

As the coordination of the production and delivery scheduling can improve the overall operational performance of the supply chains, it has being recently considered by researchers.

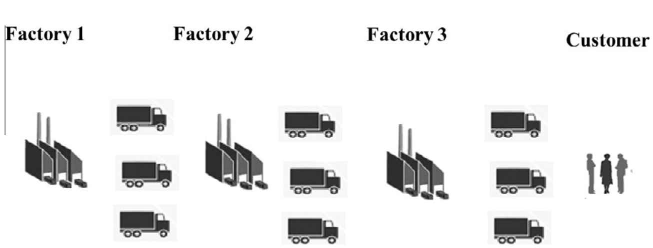

Fig. 1. Serial multi-factory supply chain.

Single machine scheduling problems with non-identical job release times and delivery times has been studied by Potts (1980) and Hall and Shmoys (1992). A sufficient number of vehicles are considered to be available to deliver jobs. Herrmann and Lee (1993) considered a single machine scheduling problem when the due dates of jobs are common. They minimized the summation of earliness–tardiness and batch delivery costs where the cost per tardy delivery is fixed. Cheng, Gordon, and Kovalyov (1996) considered a problem of the delivery costs and the summation of earliness of the orders for single machine. They also established a relation between this problem and parallel machine scheduling. They extended the complexity results and algorithms of the parallel machine scheduling problem to their problem. Hall and Potts (2003) considered number of scheduling, batching and delivery problems in the context of a two-stage supply chain where a supplier makes deliveries to several manufacturers, who also make deliveries to customers. They minimized the summation of scheduling and delivery cost. Capacity restrictions on delivery batches are considered on the studies of Lee and Chen (2001), Chang and Lee (2004), Potts (1980), Hall and Shmoys (1992) and Li, Vairaktarakis, and Lee (2005). In these studies only some scheduling objectives like maximum completion time (makespan), summation of completion times, maximum lateness, number of tardy jobs and total tardiness, are considered without taking into account any delivery costs. A comprehensive literature review on production and distribution scheduling models' was presented by Chen (2010). The problem of scheduling and batch delivery to a customer with the aim of minimizing the summation of the total weighted flow time and delivery cost on a single machine is considered by Mahdavi-Mazdeh, Shashaani, Ashouri, and Hindi (2011). Mahdavi-Mazdeh, Sarhadi, and Hindi (2007) and Mahdavi-Mazdeh, Sarhadi, and Hindi (2008) have also minimized the summation of the total flow time and delivery cost considering multiple customers with zero and non-zero ready time. An integrated due date assignment and single machine production and batch delivery scheduling problem for make-to-order production system is addressed by Rasti-barzoki and Hejazi (2013). In their research, manufacturer received number of orders from customer and this orders need to be processed on one or two machines and finally to be sent to the customer in batches. Their goal was minimizing the summation of the total weighted number of tardy jobs and the delivery costs.

Thus, the objective of this paper is to design the multi-factory supply chain to simultaneous assessment of both prementioned costs. One of the manifest differences of this problem with flowshop is transportation among factories and considering its cost as an objective function; because in the flowshop system the transportation among machines is usually negligible. Another

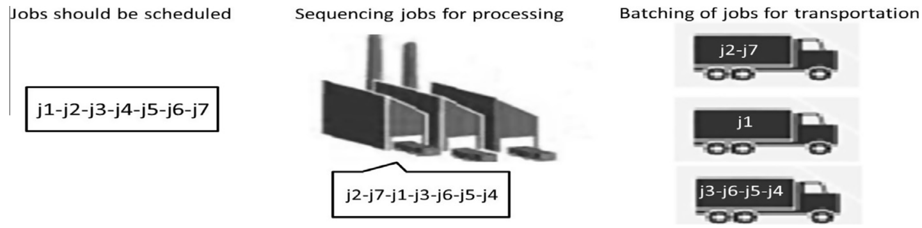

Fig. 2. Sequencing and batching of the jobs in the supply chain.

distinctive feature of this problem from the flow shop is the kind of transportation. The coordination of production and transportation is a challenging problem for this collaborative environment. Designing a batch of products, to be delivered, is a reasonable approach in reducing transportation cost. But, from the other point of view, it may increase the number of tardy jobs or the total tardiness. Thus, the goal of the problem is to determine simultaneously the optimal number of batches, the assignment of jobs to the batches and the scheduling of batch in a way that minimize the problem's objective function. As indicated from the literature, most of the multi-factory scheduling problems disregarded serial systems. Furthermore, interrelations among factories have not been considered yet. There exists no study which investigates the batch delivery of the product among factories which significantly requires designing a tradeoff between transportation and scheduling. One of the advantages of considering this holistic view was investigated in our previous work and compared with considering each factory as separate individuals in the supply chain. Here a branch and bound approach is presented for attaining exact solution. But, as the problem is NP-hard, this approach is capable of solving the problem for a limited number of jobs. Thus it is applied for small and medium size of the problem here. Based on Chung, Flynn, and Kirca (2006) a lower bound is also presented for this problem to speed up the convergence of the method.

The rest of the paper is organized as follows. Section 2 presents description of the problem. Section 3 presents a branch and bound solution approach for the problem. Computational analysis of the branch and bound method is reported in Section 4. Finally, the conclusions and future research works are drawn in Section 5.

2. Problem definition

2.1. Assumption and notations

This study investigates the multi-factory supply chain scheduling problem. Factories are designed in this supply chain in serial order. There are n jobs to be processed in the system. Final product of each factory should be delivered to the downstream factories as its raw material. In order to decrease the transportation cost of jobs, they are allowed to be delivered in batches. The batch completion time is equal to the completion time of the last job in the batch. Jobs are expected to be delivered to the customer before their predetermined due dates. Tardiness cost is incurred job which is delivered to the customer after its due date. Considering number of jobs in a batch would cause the minimization in total transportation cost. But, as jobs should be maintained in each factory until the completion time of the batch, the tardiness cost of jobs may increase. Therefore, the design of the batch is one of the main issues to be considered in this paper which can bring a compromise in tardiness and transportation costs. Thus goal of this paper is determining the number of batches, assignment of each job to a batch and also production and delivery scheduling of batches in each factory to minimize the summation of total tardiness and total transportation costs. Where values of tardiness and transportation costs are calculated as below.

The tardiness value of each job is calculated as below:

$$T_j = (A_h - d_j) - M \times (1 - \sigma_{jh})$$

Thus the value of total tardiness can be estimated by $\sum_{j=1}^{n} T_j$. The transportation cost is a function of number of batch deliveries. The number of batches transported among the system can be calculated by $\sum_{h=1}^{n} \Delta_h$.

Hence the objective function of each solution of the problem is:

$$\text{Minimize} \quad Z = \eta \times \sum_{j=1}^{n} T_j + \beta \times \sum_{h=1}^{n} \Delta_h.$$

It is assumed that:

- All jobs are available at zero time.
- Job processing in each factory cannot be interrupted.
- Factories are always available, with no breakdowns or scheduled/unscheduled maintenance.
- Infinite buffer exists between factories, before the first and after the last factory.
- Setup times are negligible.
- Jobs are available for processing in a factory immediately after arriving to it.
- Each factory can process at most one job at a time.
- A job cannot be processed in more than one factory at the same time.
- Number of jobs in each batch is at most equal to the batch size (vehicle capacity).
- Transportation times between factories are considered.
- Jobs are available for transferring between factories immediately after completion of processing the last job in the batch in the previous factory.
- There are sufficient number of vehicles for transportation.
- All data are known deterministically.
- There is no limitation on the number of batches.

By considering single machine in each factory and neglecting the transportation time between factories and also considering a single job per transportation instead of batch of jobs, the problem can be simplified to a typical flowshop scheduling problem. As Rinnooy (1976) proved that a flowshop scheduling problem is NP-hard, the mentioned problem is NP-hard too.

3. Branch and bound method

In this section the focus is on the design of an exact method which finds an optimal solution for the mentioned problem. Here in order to search the solution space, a branch and bound (B&B) method is recommended which exploits a tree structure. Each node of the tree is representative of a partial schedule and describes adding the new job to the sequence and its batching condition. Jobs are added in each node based on the earliest modified due date value of jobs (which will be described in the next section). The proposed B&B has applied the depth first search strategy for the search tree. It will be continued until no more jobs remain to be added to the partial schedule, thus the method consists of n levels.

3.1. Heuristic

The initial step of the B&B method is calculating a reasonable upper bound of solutions which would enhance the methods performance and accelerate the speed of the method. This can be done by implementing a heuristic which provides a good solution in a reasonable time. The proposed heuristic can also be used as a standalone method. This heuristic procedure obtains a batching of jobs and number of batches needed and also the scheduling of these batches, as shown in Fig3.

As mentioned earlier the objective function of the paper is the summation of transportation cost and total tardiness cost.

Assume a problem with 4 jobs to be processed. The upper bound procedure can be depicted as follows (the cost values in each step are assumptive to better describe the procedure):

- Consider the order of jobs according to their modified due date value are as: 4,2,1,3.
- Constitute the current solution by considering each job as a separate batch. Assume the cost value is equal to 200.

4	2	1	3	Cost=200

- Check if it is beneficial to join the job 2 to the first batch.

4,2	1	3	Cost=250

- As it has the worse cost value than the current solution, continue with the current solution and ignore this solution.
- Check joining the job 1 to its previous batch.

4	2,1	3	Cost=180

- Its cost value is better than the current solution. So replace it by the current solution and continue with this solution.
- Check joining the job 3 to its previous batch.
- As the batch size is 2 it can not join the previous batch.
- The obtained solution is :

4	2,1	3	Cost=180

3.2. Branching schemes

The sequences of the first to the last job are generated through the branching procedure. In general, a number of branches are generated for each node. Each node in level k is representative of a partial sequence (σ) in which the sequence of k jobs is determined and $n - k$ jobs are in the un-sequenced set and should be sequenced after these sequenced jobs. At each node, some children nodes are created. In each child node, a job which is not in the σ would be added to the end of this sequence. As the batching process should be considered at this study, two branches should be generated for each unscheduled job at each node; in one branch, the new job should be added to the last existing batch and in the other branch the new job constitute a new single-job batch (this is demonstrated in Fig4). Thus in the level k the number of generated branches are $2 \times (n - k)$.

For instance, the branching scheme of a simple problem with 4 jobs is depicted below. Assume that their order based on modified due date is 4, 2, 1, 3. The label of each node represents the sequenced job and their batching condition. The numbers which are in the same bracket are considered in the same batch. The node with the label [4][2,3] means that the job sequencing order is 4, 2, 3 and job 4 is in single-job batch and jobs 2, 3 are in another batch. In the node [4][2], four branches are generated: [4][2,1], [4][2][1], [4][2,3] and [4][2][3]. Fig4 demonstrate the branching scheme of this example.

3.3. Lower bounding schemes

Lower bound value should be calculated in each node of the decision tree. The efficient lower bound would significantly reduce the time and efforts needed for the B&B method.

Based on the main feature of the problem, the lower bound value for the problem is the summation of lower bounds on the total tardiness cost and the transportation cost. The lower bound on the cost of each partial schedule at each node (LBC) can be derived as follows.

$$\mathrm{LBC} = \eta \times \mathrm{LBT} + \beta \times \mathrm{LBD}$$

where LBT is the lower bound on the total tardiness cost, LBD is the lower bound on the transportation cost.

Proposition 3.3.1. The lower bound on the total tardiness is equal to total tardiness obtained in the schedule formed considering each batch as a single-job one.

Proof. If the batch contains more than one job, completed jobs of a batch should remain in the factory until finishing processing of uncompleted jobs of the same batch. Thus, this would results in the increment in the value of total tardiness. Thus this effect can be prevented by forming each batch by a single job. □

Proposition 3.3.2. Inspiring from Chung et al. (2006), lower bound for the total tardiness value of a node with partial sequence σ, is as below.The following notations are considered in defining the lower bound:

Step 1: Jobs should be sorted in the ascending order of their modified due date value. The modified due date value of jobs can be calculated as:

$$md_j = d_j - \left(\sum_{f=1}^{F} p_{jf} + \sum_{f=1}^{F} \tau^f\right)$$

Step 2: Consider each sorted job as a separate batch (the initial number of batches is equal to total number of jobs). Assume this schedule as the current schedule (Ω)

Step 3: Set $k = n$

Step 4: For $s = 1$ to k do

- $y = s + 1$ to k
- if size of the current batch (s) is less than the predetermined batch_size (B)
  - Move the job y to batch s and consider this schedule as (Ω')
  - if it improves the objective function value replace it with the current solution ($\Omega = \Omega'$)
- $k = k - 1$

Step 5: Stop

Fig. 3. Heuristic procedure.

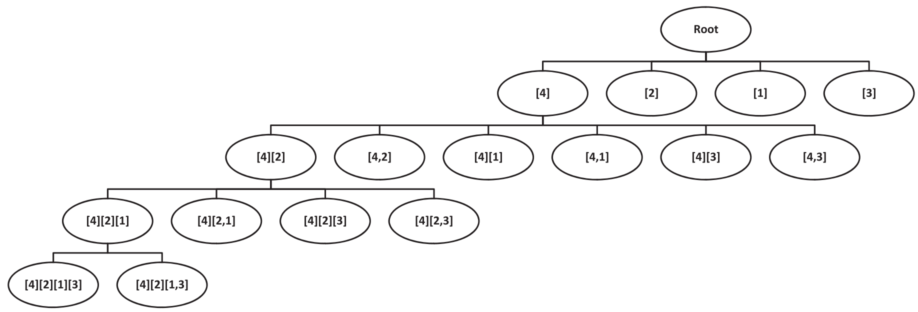

Fig. 4. Branching scheme of an example.

- $L_f(\sigma)$: the completion time of the last job in the σ on the factory f.
- $b\_s(\sigma)$: number of batches of the partial schedule.
- $n\_b(\sigma)$: number of jobs of the last batch in the partial schedule.
- $C\_b(\sigma)$: remained capacity of the last batch in the partial schedule $(B-(n\_b(\sigma)))$.
- $TT(\sigma)$: the total tardiness of jobs of the partial schedule σ.
- S: the number of jobs in the partial schedule σ.
- Us: the set of job are not included in the partial schedule σ.
- $p_{[j]f}$: the jth smallest value of the p_{jf}.
- E_{jf}: earliest start time of job j in factory f.
- $E_{[j]f}$: the jth smallest value of the E_{jf}.
- F_{jf}: earliest finish time of job j in factory f.
- $F_{[j]f}$: the jth smallest value of the F_{if}: $\{i \in Us\}$.
- $t_{[j]kl}$: the jth smallest value of the t_{ikl}: $\{i \in Us\}$.
- $r_{[j]f}$: the jth smallest value of the r_{if}: $\{i \in Us\}$.
- l_{jf}: the lateness value of the job j in factory f.
- $l_{[j]f}$: the jth smallest value of the l_{if}: $\{i \in Us\}$

where $j = 1,\ldots,n-s$, $f = 1,\ldots,F$. Mathematical definition of t_{ikl} and r_{if} are presented in the following.

To calculate the lower bound, following steps are required.

Earliest start and finish time for the first unscheduled job in the system can be calculated as below:

$$E_{[s+1]1} = L_1(\sigma)$$

$$F_{[s+1]1} = E_{[s+1]1} + p_{[s+1]1}$$

$$E_{[s+1]f} = F_{[s+1]f-1} + \tau^{f-1}$$

$$F_{[s+1]f} = E_{[s+1]f} + p_{[s+1]f}$$

where $f = 2,\ldots,F$.

Earliest start and finish time for the other unscheduled jobs in the system are defined as:

$$E_{[j]1} = F_{[j-1]1}$$

$$F_{[j]1} = E_{[j]1} + p_{[j]1}$$

$$E_{[j]f} = \max\left(F_{[j]f-1} + \tau^{f-1}, F_{[j-1]f}\right)$$

$$F_{[j]f} = E_{[j]f} + p_{[j]f}$$

where $j = s+2,\ldots,n$ and $f = 2,\ldots,F$.

As this is a factory-based lower bound, the tardiness value of each job on each factory can be calculated through the following equations:

$$t_{jkl} = \sum_{m=k}^{l} p_{jm} + \sum_{m=k}^{l} \tau^m$$

$$r_{jf} = d_j - t_{jfF}$$

$$l_{[j]f} = E_{jf} - r_{[j]f}$$

$$Tr_f = \sum_{j=1}^{n} l_{[j]f}^{+}$$

where $j = s+2,\ldots,n$ and $f = 1,\ldots,F$.

Thus the lower bound on the tardiness value with partial sequence σ is as bellows:

$$LBT = TT(\sigma) + \max_{f=1}^{F}\{Tr_f\}$$

Proof. Simply it can be understood that the start time of job j on factory f is not greater than E_{jf}. Thus the lateness of each job on each factory is not less than the $E_{jf} - r_{[j]f}$. As the value of E_{jf} are in the non-decreasing order $(E_{1f} \leqslant E_{2f} \leqslant E_{3f} \leqslant \cdots \leqslant E_{nf})$, from the preposition below, the value of l_{jf} is a lower bound for every sequence of jobs. □

Proposition 3.3.3 Kim (1995). Let R_i and d_i be nonnegative real numbers associated with job i for $i = \{1,2,\ldots,n\}$, and δ be a sequence of jobs, and let $\delta(i)$ denote the ith job of the sequence δ. Let δ_1 be a sequence in which the jobs are ordered according to the EDD rule. Then $d_{\delta_1(i)} \leqslant d_{\delta_1(i+1)}$ for $i = \{1,2,\ldots,n-1\}$. If $R_1 \leqslant R_2 \leqslant \cdots \leqslant R_n$, then for any sequence δ:

$$Z \equiv \sum_{i=1}^{n}(R_i - d_{\delta(i)})^{+} \geqslant \sum_{i=1}^{n}(R_i - d_{\delta_1(i)})^{+}$$

Proposition 3.3.4. The lower bound in the delivery cost would be achieved by the schedule formed considering each batch as a full batch. Thus the number of batches is at least equal to:

$$\mathrm{LBD} = b\_s(\sigma) + \left\lceil \frac{Us - C\_b(\sigma)}{B} \right\rceil$$

Proof. Considering number of jobs less than the batch size in each batch would certainly cause a lower tardiness cost but on the other hand would increase the cost of delivery. Filling the batch with the number of jobs equal to the batch size would avoid this cost. In this formulation, firstly unscheduled jobs are assigned to the last existing batch to fill it. Then, the remained jobs are divided to batches in the manner that fill each batch. □

3.4. Pruning schemes

The search procedure in one branch may be terminated due to one of the following conditions:

- The current node is a leaf; in other words, all jobs are sequenced.

Table 1
The comparison of B&B and GAMS.

Number of jobs	Number of factories	GAMS			B&B		
		Cost	Time	Number of nodes	Cost	Time	Number of nodes
3	3	4010	0.03	6	4010	0.015	6
	4	11,430	0.03	10	11,430	0.018	10
	5	19,070	0.09	11	19,070	0.019	10
	6	22,420	0.13	12	22,420	0.023	10
	3	8180	0.05	57	8180	0.018	21
4	4	16,660	0.09	58	16,660	0.019	17
	5	27,030	0.14	68	27,030	0.021	19
	6	31,460	0.15	79	31,460	0.025	19
5	3	11,380	0.09	196	11,380	0.023	27
	4	21,780	0.11	243	21,780	0.025	29
	5	34,650	0.11	171	34,650	0.025	33
	6	40,310	0.14	182	40,310	0.034	35

Table 2
The B&B results when $\eta = \beta$.

Number of jobs	Number of factories	B&B		B&B without upper bound		B&B without lower bound	
		Running time	Number of nodes	Running time	Number of nodes	Running time	Number of nodes
3	3	0.017333333	10	0.023666667	11	0.016333333	18
	5	0.015333333	10	0.012	11	0.010333333	18
	8	0.014333333	11	0.012666667	12	0.013333333	18
5	3	0.021666667	34.33333333	0.017	43	0.016	821
	5	0.020333333	34	0.017333333	42	0.018	821
	8	0.018666667	36	0.024	75	0.018666667	821
10	3	0.040333333	917	0.041	1409	587.1315	17,471,838
	5	0.058666667	845	0.071	1128	699.9345	17,471,845
	8	0.080666667	807	0.127	1719	848.968	17,471,848
15	3	0.715	19,728	0.950666667	28,023	*	*
	5	1.921333333	24,111	2.664	33,362	*	*
	8	12.29866667	75,870	18.04766667	116,423	*	*
20	3	34.44666667	631,656	41.477	678,867	*	*
	5	38.41066667	268,072	55.17566667	331,261	*	*
	8	1194.140667	3,928,086	1711.107	5,157,360	*	*

- The corresponding lower bound of the current node is equal or greater than the upper bound.
- The node is forbidden, i.e. number of jobs assigned to a batch is greater than the pre-determined size of the batch.

After termination of search in one branch and thus closing the node, the procedure would be continued for the last unvisited node in the previous level. If all the nodes are visited in the B&B method, the current upper bound would be considered as the optimal solution of the problem.

4. Computational results

In order to test the B&B method for the mentioned problem, extensive computational experiments are performed. The method is coded in C++ software and executed on a PC with Intel Core 2 Duo and 2 GB of RAM memory.

For this experiments, due date of each job can be generated through the adapted method of Ruiz and Stutzle (2006):

- The total processing time of each job ($T \setminus P_j$) can be calculated as:

$$T\_P_j = \sum_{f=1}^{F} P_{jf}$$

- The total transportation time can be calculated as:

$$T\_\tau = \sum_{f=1}^{F} \tau^f$$

- Define the due date of each job:

$$d_j = (T\_P_j + T\_\tau) \times (1 + 3 \times rand)$$

where *rand* is a uniformly distributed random number [0,1] and processing times of each job in each factory would be generated from the discrete uniform distribution with a range of [1,99].

Firstly, in order to assess the efficiency of the B&B and to show its computational advantages, a comparison with GAMS software is performed. Because of the high complexity of the problem and limitations of computers, GAMS is capable of solving small size instances. For this experiment, instances with 3–5 jobs and 3–6 factories are taking into accounts and other parameters are calculated from the descriptions above. Delivery cost and tardiness cost of each batch are assumed to be equal to 10 and batch size equal to 2. GAMS 24.1 is implemented for this problem, using CPLEX solver.

The table above (Table 1) shows that the quality of the solution (cost value) found by B&B method is the same as the GAMS method. But the GAMS computational time and searched nodes has the considerable larger value than the B&B. Thus through this table, it is proved that the B&B is more efficient than GAMS.

The efficiency of the B&B method is investigated on several problems involving up to 20 jobs and 8 factories. These test problems are generated according to the following parameters:

- Number of jobs (n): 3, 5, 10, 15, 20.
- Number of factories (F): 3, 5, 8.
- Delivery cost of each batch: 10, 1000.
- Tardiness cost a job: 10, 1000.
- Batch size: 2.

Table 3
The B&B results when $\eta < \beta$.

Number of jobs	Number of factories	B&B		B&B without upper bound		B&B without lower bound	
		Running time	Number of nodes	Running time	Number of nodes	Running time	Number of nodes
3	3	0.012666667	13	0.011	14	0.012666667	18
	5	0.011	13	0.012	14	0.012333333	18
	8	0.013333333	13	0.008	14	0.014	18
5	3	0.015666667	138	0.013	152	0.014	821
	5	0.011	72	0.013	88	0.016333333	821
	8	0.017333333	78	0.013	139	0.016333333	821
10	3	1.691666667	185,682	1.821	198,734	585.513	174,718,148
	5	0.923333333	46,780	1.015666667	53,284	760.3165	174,718,148
	8	1.883666667	40,676	2.210333333	57,979	1008.4085	174,718,148
15	3	462.089	24,828,627	809.305	49,145,167	*	*
	5	486.803	12,124,739	520.922	13,470,921	*	*
	8	756.491	6,623,932	1238.288	17,882,264	*	*
20	3	*	*	*	*	*	*
	5	*	*	*	*	*	*
	8	*	*	*	*	*	*

Table 4
The B&B results when $\eta > \beta$.

Number of jobs	Number of factories	B&B		B&B without upper bound		B&B without lower bound	
		Running time	Number of nodes	Running time	Number of nodes	Running time	Number of nodes
3	3	0.013	10	0.009333333	11	0.012666667	18
	5	0.016333333	10	0.012333333	11	0.013333333	18
	8	0.011333333	11	0.013	12	0.011	18
5	3	0.012	36	0.016666667	43	0.018	821
	5	0.013	34	0.015666667	42	0.017333333	821
	8	0.013666667	36	0.015333333	75	0.018333333	821
10	3	0.03	841	0.037333333	1323	604.4745	174,718,148
	5	0.056	948	0.062666667	1083	774.019	174,718,148
	8	0.087333333	805	0.145	1672	1037.215	174,718,148
15	3	0.733333333	17,708	0.905	24,852	*	*
	5	2.232333333	21,958	3	29,426	*	*
	8	14.73133333	73,738	18.962	111,012	*	*
20	3	38.77333333	547,508	40.19333333	577,884	*	*
	5	42.087	226,583	50.49733333	292,122	*	*
	8	1419.5	3,808,964	1717.957	4,945,083	*	*

Table 5
The effect of costs on number of batches and tardiness.

Number of jobs	$\eta = \beta$		$\eta > \beta$		$\eta < \beta$	
	Number of batch	Tardiness	Number of batch	Tardiness	Number of batch	Tardiness
3	3	398	3	398	2	695
5	5	1133	5	1133	3	1637
8	8	2452	8	2452	4	2684
10	10	3840	10	3840	5	5556
15	15	7186	15	7186	8	9471
20	20	12,179	20	12,179	*	*

45 test problems are generated based on different combinations of these parameters. We arrange these test problems in 3 tables (Tables 2–4) based on the delivery cost and tardiness cost values. (The * in some cells of tables indicates that the corresponding instance could not be solved during the predetermined time limit, 7200 s.)

Through these tables the impact of the lower bound and upper bound are investigated in the performance of the proposed B&B method. Results represent that in the small size of the problem, especially for problems with 3, 5 jobs, the role of lower bound and upper bounds are not important. But for the larger size of the problem, as the number of jobs increases, the effect of these bounds become more significant in the efficiency of the method. Similar results can be driven for increase in the number of the factory.

As it is described in the problem definition, in each step of the method, it should be checked whether to fill each batch for delivery or send it when it is not full. From the Table 3 it can be deduced that when the delivery cost is greater than the tardiness cost, computation becomes so complex that it cannot find the solution even for 20 jobs in the time limit.

From the tables above, it can be concluded that the value of tardiness and delivery costs have considerable effect on computational time and searched nodes of the method.

In Table 5, effect of different values of tardiness cost and transportation cost is investigated on the number of batches and tardiness values. For this reason, some instances of problem are considered from data generated above only for number of factories equal to 3 (for other number of factories the same comparison can be done.) and batch size equal to 2. It is clear that when the value of tardiness cost is lower than the transportation cost, it is preferred to transport jobs in batches with greater number of jobs. But in the adverse situation, it is better to deliver jobs in batches with smaller number of jobs.

Table 5 shows that as the cost of delivery increases, number of the batches delivered among factories decreases and though the tardiness value of jobs grows.

5. Conclusion

This paper studied coordination between scheduling and distribution (transportation) system in a supply chain network for the first time. It considered a serial multi-factory supply chain scheduling problem in which jobs are transported among factories and finally delivered to the customer. Transportation capacity (batch size) and transportation costs are also taken into account. Designing batch of jobs for transportation in the system causes reduction in transportation costs. The objective considered in this paper is the summation of total tardiness and transportation costs. A branch and bound method for solving this problem is presented. Efficiency of this method is investigated for up to 20 jobs and 8 factories of this problem. A lower bound and a heuristic (as an upper bound) are presented to enhance the performance of the method. Investigating the effect of parameters on the performance of the method, computational experiments are presented.

There are many issues for future exploration. First of all, other optimization technique can be developed for finding optimal or near optimal solution for this problem. Real world constraints of the scheduling environments can be considered such as limited buffer capacities between factories. Models involving multiple customers with vehicle routing decisions may be promising alternatives.

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