Deployment strategy of WSN based on minimizing cost per unit area

Fan Tiegang, Teng Guifa, Huo Limin

Abstract

Unbalance energy consumption is an inherent problem in wireless sensor networks characterized by multi-hop routing and many-to-one traffic pattern. Uneven energy dissipation can waste a lot of energy and cost. In this paper, a new deployment strategy of WSN that gathers several means is proposed to minimize cost. The regular hexagonal cell architecture is employed to build network that satisfies the constraints of coverage and connectivity. Based on the analysis of energy consumption of sensors and sink and cost of network, an energy allocation theorem and an integer programming model are presented to minimize the cost per unit area. The key issue is to determine the number of layers of network when other parameters are fixed. Furthermore, a scheme of multi-sink network is proposed for large monitored area. In order to balance the energy consumption of sensors on the identical layer, a uniform load routing algorithm is presented. The numerical analysis and simulation results show that the waste of energy and cost of WSN can be effectively reduced with the strategy.

1. Introduction

Wireless Sensor Networks (WSN) are composed of hundreds of sensors connected by wireless link. Those sensors have low level of sensing, computation, storage and communication ability. They can collect surrounding information and then transmit data to sink or base station. WSN can cover a wide range of application, such as environmental and habitual monitoring, disaster managing, wildlife tracking, health care, production control, traffic management and so on.

In general, wireless sensor is battery-powered. It is usually impossible or impractical to recharge battery. So the lifetime of WSN is mainly determined by the battery life. Preserving the sensor’s energy is a key for keeping the network operational for longer periods of time. One important factor that causes the intrinsic limits of performance and system scalability in homogeneous sensor networks is the uneven energy depletion problem among sensors. The typical case is the Energy Hole Problem in multi-hot wireless sensor networks. Those sensors that are one-hop away from sink need to relay more packets from other sensors to sink. So they have much heavier traffic burden and their energy depletion is faster than other sensors. Thus these sensors may die out very early resulting to network disconnection, although there are still significant amounts of energy in the other sensors of the network. The uneven energy depletion reduces the useful lifetime of network and causes a lot of waste. It should be prevented to the largest extent. Experimental results in [1] show that, by the time the sensors one-hop away from the sink exhaust their energy budget, sensors farther away still have up to 90% of their initial energy budget if the sensors are uniformly distributed in the network. A considerable amount of energy is wasted so as to make the network cost increase greatly.

This paper focuses on the cost of network that satisfies some constraints (coverage, connectivity and lifetime). The cost of WSN consists of hardware cost and energy cost. In order to minimize the cost, we consider several aspects, such as minimum number of sensors, the minimum residual energy, the optimal hop number of data transmission, and uniform load routing, etc. A controlled node deployment strategy is provided to deploy multi-sink network. To sum up, the main contributions are presented as follows.

(1) This paper formulates the energy consumption of sensor and sink and provides an energy allocation theorem.
(2) By analyzing cost of network, an integer programming model is proposed to minimizing cost per unit area of network.
(3) A scheme of multi-sink network for large monitoring area is detailed.
(4) A uniform load routing algorithm is proposed to balance the energy consumption of sensors on the identical layer.

The rest of this paper is organized as follows. Section 2 introduces related work. In Section 3, several models to be used are given. We analyze energy consumption and cost of network and
propose an integer programming model in Section 4. A Scheme of multi-sink network is presented in Section 5. Section 6 contains some numerical analysis and simulation results. Finally the paper is concluded with some mention about the future scope of the work in Section 7.

2. Related work

In order to avoid the uneven energy depletion and reduce the waste, many works have been reported [2]. Nonuniform node distribution strategy was proposed. In [1], some pure relay nodes were added to the network to reduce other nodes traffic burden. Ref. [3] proposed a power-aware nonuniform node distribution scheme. They derived node distribution functions based on hop counts. Ref. [4] explored the theoretical aspects of the nonuniform node distribution strategy and proposed a nonuniform node distribution strategy to achieve nearly balance energy depletion in network. Some works [5–7] focused on adjusting node different transmission range. Ref. [5] presented an improved coronal model with levels for analyzing sensors with adjustable transmission ranges in a WSN with circular multi-hop deployment. Two algorithms were proposed, CETT and CETL, for assigning the transmission range of sensors in each corona for different node distribution. Ref. [6] proposed a hybrid communication mode which is a combination of single-hop and multi-hop modes, and which is more cost-effective than either of the two modes. Ref. [7] formulated the energy consumption balancing problem as an optimal transmission data distribution problem by combining the ideas of corona-based network division and mixed-routing strategy together with data aggregation and an energy-balanced data gathering protocol was designed.

To employing mobile nodes or sink for balancing the energy consumption are mentioned in some literatures. In [8], mobile relay nodes were used to prolong the lifetime of WSN. A movement-assisted data gathering scheme was proposed in [9]. Some mobile sinks that change their location when the nearby sensors energy becomes low were used to improve lifetime of network. Some works explored the nonuniform clustering algorithm to mitigate the Energy Hole Problem. Ref. [10] partitioned the nodes into clusters of unequal size. The cluster closer to sink had small size than the farther away from the sink. Thus cluster heads closer to sink can preserve some energy for the inter-cluster data forwarding. This unequal clustering mechanism balanced the energy consumption and elevated the energy efficiency. Ref. [11] showed the optimal cluster-radius that make the lifetime of network maximal by theoretical analysis. Based on the optimal cluster-radius, a rotation strategy of unequal cluster-radius was proposed to improve the lifetime of network.

Above works were implemented based on the assumption that homogenous sensors with equal battery capacities were used. Some works focused on allocating different battery capacities to sensor nodes. Ref. [12] proposed to use levels of batteries and investigated the effect multiple battery levels to maximize the useful lifetime of network. Ref. [13,14] formulated the cost-constrained heterogeneous WSN battery allocation problem as an INLP. A rapid heuristic algorithm was provided to produce near-optimal solution. Ref. [15] formulated a constrained multiple deployment problem by considering energy models of the battery energy budget and three sensor operations. There are other methods being used to balance energy consumption, such as data aggregation, appropriate data storage. Ref. [16] proposed a load storage method based on rings to balance energy consumption. Actually an effective method always integrates many schemes or algorithms. Ref. [17] found that combination of data compression and hierarchical topology of network was effective in alleviating the Energy Hole Problem.

The purpose of balancing energy consumption is to reduce energy waste, improve energy efficiency and extend the lifetime of network. From another point of view, it is to reduce the cost of network under certain conditions. But the cost of network is not concerned directly in above works. When increasing the density of nodes, large amount of redundant data have been produced. Transmission and processing of redundant data is also a waste. The node mobility also consumes additional energy. So above methods are to reduce the waste of energy in some way. However, the effect for reducing total cost of WSN is limited. To the best of our knowledge, there are little literatures that focus on the cost of network. Ref. [18] formulated an optimization problem with some constraints (connectivity, coverage, and lifetime) and gave a solution that minimized the overall cost of the heterogeneous sensor networks. Two types of hierarchical sensor networks were considered: random uniform deployment and grid deployment. However, author ensured that only critical nodes and cluster heads expired at about the same time. This meant that many other sensors had residual energy when the network unusable and cost of network was not the minimum. Ref. [19] formulated a generalized node placement optimization problem aimed at minimizing the network cost with constraints on lifetime and connectivity. The two representative scenarios of this problem were described. A two-phase approach was proposed, in which locally optimal design decisions were taken. But the authors only explored the problem of relay node placement in heterogeneous WSN, and the optimization objective was to minimize the number of relay nodes for a given deployment of sensor nodes. The cost of network was not considered. In [20], lifetime per unit cost of an event-driven linear WSN was analyzed. Numerical and simulation results were provided to study the optimal sensor placement and the optimal number of deployed sensors.

In our approach, lifetime of network is as a design requirement. The WSN includes two types of nodes, sensor and sink. For a given monitored area, we propose a controlled node deployment strategy to minimize the total cost of network under some constraints. Various means are used in the strategy, such as regular hexagonal cell architecture, different levels of battery, optimal layer number, uniform load routing, etc.

3. Preliminaries

In this section, some assumptions and used models are given. We consider a WSN with the following properties.

Two types of node are considered to deploy for WSN, sensor and sink. Sensor has the fixed sensing range and transmission radius. Each sensor periodically senses the environment and generates data. Sensors transmit their data as well as transmit data received from other nodes towards sink. The sink receives and aggregates data from sensors, then transmits the data to base station or other data receptor.

3.1. Energy model

In this model, all energy consumed by a sensor belong to one of the two classes: the consumption related to the amount of data and other consumption. The former includes the energy consumption of generating data, transmitting data and receiving data. The latter includes energy consumption of other network operations, e.g., routing, time synchronization, idle listening. We consider it as constant unrelated to the amount of data. The energy consumed per unit time by a sensor $E = e_1 \times x + e_2 \times y + e_3 \times z + e_4$, where $e_1, e_2, e_3$ and $e_4$ stand for the energy consumption of transmitting unit data, receiving unit data, generating unit data, and no relation to the amount of data respectively. The $x, y$, and $z$ are the...
corresponding amount of data. The energy consumption of sink is same roughly as sensor. The difference is that sink does not generate data, but aggregates received data. The energy consumed per unit time by a sink \( E' = e'_1 \times x + e'_2 \times y + e'_3 \times z + e'_4 \), where \( e'_i \) is energy consumption of aggregating unit data.

3.2. Cost model

Let \( C \) be the cost for a sensor and \( C' \) be the cost for a sink. The simple model of cost function for sensor is \( C = a + b \times E_i \), where \( a \) is the cost of sensor hardware (excluding the battery), \( b \) is a proportionality constant for the battery cost [18]. \( E_i \) is energy value equipped on the sensor. The cost function for sink is \( C' = a' + b \times E_i' \), where \( a' \) is the cost of sink hardware, \( E_i' \) is energy value equipped on sink.

3.3. Aggregation model

The sink can aggregate the data. The \( \phi(x) \) denotes the aggregation function. \( \phi(x) = m \times x + c \), where \( m \) is the amount of input data, \( m \) is compression ratio, \( 0 \leq m \leq 1 \). \( c \) is a constant. The values of \( m \) and \( c \) are selected according to the scenario.

3.4. Life time of WSN

The WSN should be considered dead once it can’t satisfy its monitoring requirements. In this paper, it is assumed that when any sensor uses up its energy, the coverage and connection of network are broken. The lifetime of WSN is defined as the time when any sensor exhausts energy. It is equivalent to the minimum among the lifetime of sensors, i.e., \( T = \min(t_i) \), where \( t_i \) is the lifetime of sensor \( i \). It is considered that the lifetime of WSN, as a design requirement, should be satisfied.

3.5. Notation

To enable this article be understood more easily, Table 1 summarizes the notations used in this paper.

### Table 1: Notation

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_1 )</td>
<td>Energy consumption of transmitting unit data for sensor</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>Energy consumption of receiving unit data for sensor</td>
</tr>
<tr>
<td>( e_3 )</td>
<td>Energy consumption of generating unit data for sensor</td>
</tr>
<tr>
<td>( e_4 )</td>
<td>Energy consumption of other network operations for sensor</td>
</tr>
<tr>
<td>( e'_1 )</td>
<td>Energy consumption of transmitting unit data for sink</td>
</tr>
<tr>
<td>( e'_2 )</td>
<td>Energy consumption of receiving unit data for sink</td>
</tr>
<tr>
<td>( e'_3 )</td>
<td>Energy consumption of generating unit data for sink</td>
</tr>
<tr>
<td>( e'_4 )</td>
<td>Energy consumption of other network operations for sink</td>
</tr>
<tr>
<td>( E_1 )</td>
<td>Energy consumption of all sensors per unit time</td>
</tr>
<tr>
<td>( E' )</td>
<td>Energy consumption of sink per unit time</td>
</tr>
<tr>
<td>( R_s )</td>
<td>Transmission range of sink</td>
</tr>
<tr>
<td>( R_c )</td>
<td>Sensing radius of sensor</td>
</tr>
<tr>
<td>( T )</td>
<td>Lifetime of network</td>
</tr>
<tr>
<td>( C )</td>
<td>Total cost of network</td>
</tr>
<tr>
<td>( C_s )</td>
<td>Hardware cost of network</td>
</tr>
<tr>
<td>( C_e )</td>
<td>Energy cost of network</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Cost per unit area of network</td>
</tr>
<tr>
<td>( a )</td>
<td>Cost of a sensor</td>
</tr>
<tr>
<td>( a' )</td>
<td>Cost of a sink</td>
</tr>
<tr>
<td>( b )</td>
<td>Cost per unit energy</td>
</tr>
<tr>
<td>( m )</td>
<td>Compression ratio</td>
</tr>
<tr>
<td>( N_j )</td>
<td>The node size in ( j )th layer</td>
</tr>
<tr>
<td>( k )</td>
<td>Number of layers of network</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>Area of regular hexagon</td>
</tr>
<tr>
<td>( S )</td>
<td>Area of network</td>
</tr>
<tr>
<td>( C'_j )</td>
<td>The ( j )th hexagon of ( i )th layer</td>
</tr>
<tr>
<td>( S'_j )</td>
<td>The ( j )th sensor of ( i )th layer</td>
</tr>
<tr>
<td>( M )</td>
<td>Number of battery levels</td>
</tr>
<tr>
<td>( Max_b )</td>
<td>The maximum buffer size of sink</td>
</tr>
<tr>
<td>( V )</td>
<td>Threshold of data size received by sink per unit time</td>
</tr>
<tr>
<td>( D_s )</td>
<td>Data size received by sink per unit time</td>
</tr>
<tr>
<td>( T_z )</td>
<td>Transmission range of sink</td>
</tr>
</tbody>
</table>

4. Analysis on energy consumption and cost

In this section, the regular hexagonal cell architecture is considered. Then energy consumption of node and cost of network are analyzed. A programming model is proposed.

4.1. Regular hexagonal cell architecture

The regular hexagonal cell architecture mentioned in [21–23] is adopted to satisfy the constraints of coverage and connectivity. The network’s coverage area is divided into regular hexagons. Sensors are deployed at the center of hexagons to monitor the hexagon area. Three adjacent sensors form an equilateral triangle. In [24], author proved that it is the optimal sensor coverage scheme in terms of minimizing the number of sensors used. The sink is located at the center hexagon. The sensors sense surrounding environment. Data are generated periodically and transmitted to sink in multi-hop mode. To be convenient to analyze, all sensors are divided into several layers, as shown in Fig. 1. Here \( i = 1 \) indicates the layer nearest to sink and \( i = k \) indicates the layer farthest from sink. The first layer contains six hexagons, the \( i \)th layer contains \( 6i \) hexagons. Let \( C'_j \) identifies \( j \)th hexagon of \( i \)th layer. The data are transmitted from sensors of \( i \)th layer to \((i-1)\)th layer, where \( i = 1, 2, \ldots, k \). The \( 0 \)th layer indicates the center hexagon cell where the sink is located. The radius of hexagon \( R_h \) is related to the sensing radius \( R_s \) and communication range \( R_c \) of sensor. In order to cover the whole hexagon area, \( R_h \) must satisfy the condition \( R_h \leq R_c \). The relationship between \( R_h \) and \( R_c \) must be \( R_h \leq \frac{R_c}{\sqrt{3}} \) for satisfying connectivity between neighboring sensors. To guarantee coverage and connection, the radius of hexagon is set as \( R_h = \min \{ R_s, \frac{R_c}{\sqrt{3}} \} \).

4.2. Analysis on energy consumption

It is assumed that the network contains \( k \) layers. The sink located at the center hexagon receives data from sensors of first
layer. The sensors of ith layer transmit the data that their own and received from the sensors of (i + 1)th layer to the sensors of (i-1)th layer, where i = 2, ..., k - 1. Only the sensors of kth layer that do not receive data transmit their own data. Because the data size received and transmitted by sensors in different layers is not same, the energy consumption of sensors in different layers are different. According to the energy model described in above section, the energy consumption of a sensor in kth layer per unit time is

$$E_k = (e_1 + e_2)L + e_4$$

(1)

The energy consumption of a sensor in ith layer per unit time is

$$E_i = e_1 t_i + e_2 r_i + e_3 L + e_4, \quad k > i \geq 1$$

(2)

where $t_i$ is the data size transmitted per unit time, $r_i$ is the data size received per unit time, $-t_i = r_i + L, r_i = \frac{n}{\pi \sum_{j=i+1}^{N} N_j} L$. This means the sensors of ith layer receive all data generated by sensors of jth layer, where j is from i + 1 to k. $N_i$ is the number of sensors in ith layer, $N_i = 6i$. So

$$r_i = \frac{\sum_{i=1}^{k} N_j}{N_i} = \frac{(k + i + 1)(k - i)}{2i} L$$

(3)

According to Section 7

$$E_i = e_1 \left[ \frac{(k + i + 1)(k - i)}{2i} L + L \right] + e_2 \frac{(k + i + 1)(k - i)}{2i} L + e_3 L + e_4$$

$$= e_1 \left( \frac{2k^2 - i^2 + k - iL + L}{2i} \right) + e_2 \frac{2k^2 + k - i^2 - iL + e_3 L + e_4}{2i}$$

(4)

From the above expression, it is observed that the energy consumption by a sensor of ith layer depends on the i when k is constant. The energy consumption increases with the decrease of i. The sensors of first layer consume maximum energy and sensors of kth layer consume least energy. This is the reason of Energy Hole Problem in homogenous sensors network. When the sensors of first layer exhaust energy, WSN is invalid. A large amount of energy of sensors in other layers left to be wasted. The ideal situation is that all sensors exhaust energy at same time. The scheme that different levels of batteries are equipped to sensors in different layers is a good way to reach the situation. It can minimize the residual energy and reduce the network cost. Some works [12–14] can be adopted to allocate different energy to sensors. Next the energy consumption of all sensors per unit time is calculated.

$$E_s = \sum_{i=1}^{k} E_i$$

$$= \sum_{i=1}^{k} \left[ e_1 \left( \frac{2k^2 - i^2 + k - iL + L}{2i} \right) + e_2 \frac{2k^2 + k - i^2 - iL + e_3 L + e_4}{2i} \right] 6i$$

$$= \left( e_1 + e_2 \right) 3L \left[ \frac{k^2}{2} - \frac{1}{2i} \right] k(k+1) + \frac{1}{6} k(k+1)(2k+1)$$

$$+ \left( 6Le_1 + 6e_3 L + 6e_4 \right) \frac{k(k+1)}{2}$$

$$= 2\left( e_1 + e_2 \right) k^2(2k + 1) L + (3Le_1 + 3Le_3 + 3e_4) k(k+1)$$

$$= 2k^3(e_1 + e_2) + 3k^2(Le_1 + e_3 L + e_4)$$

$$+ k(Le L + 2e_2 L + 3e_3 L + 3e_4)$$

(5)

From above expression, $E_i$ is a polynomial of degree 3 about k. This means that, as k increases, energy consumption of all sensors per unit time increases with cube of k. The energy consumption for transmission and reception increases more quickly than others.

All data are transmitted to sink. The energy consumption of sink includes receiving, aggregating, sending data and others (such as idle listening, time synchronization). Data size received per unit time by sink is

$$r = L \sum_{i=1}^{k} 6i = 3L(k^2 + k)$$

(6)

According to the aggregation model at above section, the data size of per unit time sent by sink is

$$s = mr + c = 3mL(k^2 + k) + c$$

(7)

The energy consumption of sink per unit time is

$$E = e_1 s + e_2 r + e_3 r + e_4$$

$$= 3Le_1^2 + e_4^2 + 3LmL(k^2 + k) + c + e_4$$

$$= 3k^2(Le_1^2 + e_4^2 + me_4) + 3kL(e_3^2 + e_4^2 + me_4) + e_1^2 c + e_4^2$$

(8)

The energy consumption of network per unit time is

$$E = E_i + E_s$$

$$= k^2 2(e_1 + e_2) L + k^2(3Le_1 + 3Le_3 + 3e_4 + 3Le_3^2 + 3Le_3e_3 + k(e_1 L - 2e_2 L + 3e_3 L + 3e_4 + 3Le_3^2 + 3Le_3e_3 + e_4^2) c + e_4^2$$

(9)

According to above analysis, it is easy to prove the following theorem.

Theorem 1 (Energy allocation theorem). Employing the network with regular hexagonal cell architecture, let T be the design lifetime of network. The energy waste of network is smallest if and only if the energy equipped on each sensor of ith layer is $E_i$ and the energy equipped on sink is $E_s$.

Proof. Wasted energy is the residual energy of all nodes. The residual energy is caused due to the different node failure time. When any node runs out of its energy, the network is dead. The residual energy of the other nodes is wasted. Therefore, wasted energy is minimized if and only if all nodes simultaneously or nearly simultaneously use up their energy. According to the previous analysis of the node energy consumption, if the energy equipped on each sensor of ith layer is $E_i$ and the energy equipped on sink is $E_s$, the lifetime of all nodes is T. This reduces the energy waste to the least. □

4.3. Analysis of cost

The cost of network includes the hardware cost of sensors and sink and cost of energy. The number of sensors is

$$N = \sum_{i=1}^{k} 6i = 3k(k+1)$$

(10)

The cost of hardware is

$$C_1 = aN + a^*$$

(11)

where $a$ is the cost of one sensor, $a^*$ is the cost of one sink. The cost of all energy is

$$C_2 = bET.$$  

(12)

where $b$ is the cost per unit energy. The total cost of network is

$$C = C_1 + C_2 = aN + a^* + bET$$

$$= 3a(k+1)k + a^* + bET$$

(13)

From expression (9) and (13), it is observed that cost of network is a polynomial of degree 3 about k, as k increases, $C$ increases with cube of k. So k is an important factor affecting the cost. The cost of network is related to monitored area. Two costs corresponding to two different area networks are not comparable. So the Cost Per
Unit Area (CPUA) is minimized to obtain $k$. Next, the CPUA is calculated. The area of regular hexagon is $S_h = \frac{\sqrt{3}}{2}a^2$. So the area of network is

$$S = S_h(N + 1) = \frac{3}{2} \sqrt{3} R_h^2 (3k^2 + 3k + 1)$$  \hspace{1cm} (14)

The CPUA is

$$\bar{C} = \frac{C_1}{S} = \frac{C_1}{S} + \frac{C_2}{S}$$  \hspace{1cm} (15)

where

$$\frac{C_1}{S} = \frac{a}{S_h} + \frac{a' - a}{S_h[3(k + 1)k + 1]}$$  \hspace{1cm} (16)

$$\frac{C_2}{S} = \frac{bT}{S} - \frac{bT}{S} \frac{E}{S} \frac{3(k + 1)k}{S_h}$$  \hspace{1cm} (17)

The expression (16) denotes that the hardware cost per unit area includes two terms. The constant term, $\frac{a}{S_h}$, denotes the sensor hardware cost per unit area. The second term is sink hardware cost per unit area and decreases as $k$ increases. The reason is easy to understand. All sensors are evenly deployed around only one sink. The expression (17) denotes energy cost per unit area. In order to facilitate analysis, an approximation is applied.

$$\frac{C_2}{S} \approx \frac{bT}{S} \frac{E}{S} \frac{3(k + 1)k}{3}$$

$$= \frac{bT}{S} \frac{e_1 L (2k + 1) + \frac{2}{3} e_2 L (k - 1) + e_3 + e_4}{S_h}$$

$$+(e_3 + e_2 + me_4) L + \frac{e_3}{3k(k + 1)} + \frac{e_4}{3k(k + 1)}$$

The operational process is omitted. From the expression, some results can be obtained. The energy cost of sensor for receiving and transmitting data per unit area increases as $k$ increases. The energy cost of sink unrelated to the amount of data decreases as $k$ increases. The energy cost of sink for receiving and aggregating data per unit area are no relational with $k$.

In summary, as $k$ increases, sensor cost per unit area including hardware and energy increases, and sink cost per unit area including both decreases. So there must a value of $k$ to minimize CPUA.

### 4.4. Programming model to minimize CPUA

The objective function to be minimized is CPUA. We consider that some constraints must be met. It is assumed that the number of battery levels that can be used is $M$. Since the SNs in same layer consume the same amount of energy, the number of layers $k$ should be less than or equal to $M$.

The sink receives data from all SNs. So the buffer size of sink should be considered. Total data size received by sink per unit time must be less than or equal to a threshold ($V$) that is determined by maximum buffer size ($Max_b$), data compression ratio ($m$) and data transmission cycle ($T_c$) of sink. The threshold can be represented as a function.

$$V = f(Max_b, m, T_c)$$

We can derive a simple expression of the function

$$(mV + c) T_c = Max_b$$

$$\Rightarrow V = \frac{1}{m} \left( \frac{Max_b}{T_c} - c \right)$$

Data size received by sink per unit time ($Ds$) is

$$Ds = \sum_{i=1}^{k} \delta i = L(3k^2 + 3k)$$

Sink sends the collected data to a data receiving station outside the network. So the radius of the network should be less than the transmission distance of sink. The radius of the network ($R_h$) is

$$R_h = \left( k + \frac{1}{2} \right) \sqrt{3} R_h$$

The integer programming model is as follows

$$\text{min } \bar{C}$$

subject to

$$k \leq M$$

$$Ds \leq V$$

$$R_h \leq \alpha T_c$$

$k$ is positive integer

The inequality (19) represents the constraint of the number of battery levels. In the constraint of buffer size indicated by inequality (20), the threshold $V$ is less than or equal to maximum buffer size of sink. Equality can hold when data compression ratio $m = 1$ and data transmission cycle of sink is same as sensor’s. The inequality (21) is the constraint of transmission range of sink, where $T_c$ is the maximum transmission range of sink, and $\alpha (0 < \alpha \leq 1)$ is a parameter. The $\alpha$ is related to the position of the data receiving station. This constraint makes sure that the collected data of sink can be transmitted to data receiving station. The model can be solved by using tool software, such as Matlab, Lingo.

Next let us discuss the sink. In general, sink has larger capacity of data procession than sensor, so $a' > a$. If the sink is equipped with solar cell or solar panels, the energy consumption of sink, $E_c$, can be omitted when $\bar{C}$ is calculated. This is a special case of the above model.

After model is solved, area of network is obtained. We define the area as Minimum Cost Area of Network (MCAN). When the area that needs to be monitored is larger than MCAN, a multi-sink network can be deployed. The details are in the next section.

### 5. Scheme of multi-sink network

In the previous section, we discuss how to deploy single-sink network based on minimizing the CPUA. Note that the area of single-sink network is invariant when $k$ and $R_h$ are fixed. When the area that needs to be monitored is larger than MCAN, a multi-sink network that contains several single-sink subnetworks can be deployed. Fig. 2 shows a multi-sink network that contains five single-sink subnetworks. It is assumed that sink can send data far enough directly to the data receiving station. The scheme of deploying multi-sink network is as follows.

1. The $k$ and $R_h$ can be got according to previous sections. The shape of single-sink network is similar as a regular hexagon. Its radius is $R_h = (k + \frac{1}{2}) \sqrt{3} R_h$.

2. The scale that needs to be monitored is divided into regular hexagons of radius $R_h$. A single-sink subnetwork will be built in each hexagon. The division is relative to the shape of area monitored. For simplicity, it is assumed that the division can be completed.

3. Each regular hexagon is divided into smaller regular hexagons with radius $R_h$. Sensors and sink are deployed at the centers of small hexagons. The following steps are done to obtain those centers.

Step 1. Drawing three diagonals of the hexagon to connect the corresponding vertices, the intersection point of diago-
6. Numerical analysis and simulation results

In this section, the numerical analysis and simulation results of the single-sink network and multi-sink network are presented. Existing network simulators are inadequate for simulating network of hundreds of sensors with lifetime on the order of several months. The common problem of these simulators is their relatively high level of detail on the physical and MAC layers. So, a dedicated simulator has been written that abstracts many of lower layer issues and focuses on the energy consumption problem. All calculations and simulation in this section are completed by Matlab.

6.1. Simulation environment

It is assumed that simulations are based on a collision-free MAC protocol without data loss and sink can send directly data to data receiving station. For aggregation model, the data compression ratio of sink varies from 0 to 1, c = 0. For energy consumption model, the parameters are set as follows: \( e_1 = 10^2 \text{nJ/bit} \), \( e_2 = 100 \text{nJ/bit} \), \( e_3 = 10^3 \text{nJ/bit} \), \( e_4 = 10^4 \text{nJ/bit} \), \( e'_1 = 2 \times 10^3 \text{nJ/bit} \), \( e'_2 = 3 \times 10^3 \text{nJ/bit} \), \( e'_3 = 10^4 \text{nJ/bit} \), \( e'_4 = 10^5 \text{nJ/bit} \). For cost model, \( a = 20 \text{s} \), \( a' = 40000 \text{s} \), \( b = 25 \text{$/J} \). Table 2 lists the parameters in detail.

### Table 2

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network</td>
<td>Radius of regular hexagon ( (S_h) )</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>The parameters ( (x) )</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Data generation rate ( (L) )</td>
<td>200 bits/sensor minute</td>
</tr>
<tr>
<td>Aggregation</td>
<td>compression ratio ( (m) )</td>
<td>( 0 \sim 1 )</td>
</tr>
<tr>
<td></td>
<td>Consent ( c )</td>
<td>0bits</td>
</tr>
<tr>
<td>Energy</td>
<td>( e_1 )</td>
<td>( 10^2 \text{nJ/bit} )</td>
</tr>
<tr>
<td></td>
<td>( e_2 )</td>
<td>100 nJ/bit</td>
</tr>
<tr>
<td></td>
<td>( e_3 )</td>
<td>50 nJ/bit</td>
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<tr>
<td></td>
<td>( e_4 )</td>
<td>10 nJ/bit</td>
</tr>
<tr>
<td></td>
<td>( e'_1 )</td>
<td>( 10^4 \text{nJ/bit} )</td>
</tr>
<tr>
<td></td>
<td>( e'_2 )</td>
<td>( 3 \times 10^3 \text{nJ/bit} )</td>
</tr>
<tr>
<td></td>
<td>( e'_3 )</td>
<td>( 2 \times 10^4 \text{nJ/bit} )</td>
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<tr>
<td></td>
<td>( e'_4 )</td>
<td>( 10^5 \text{nJ/bit} )</td>
</tr>
<tr>
<td>Cost</td>
<td>Cost of a sensor ( (a) )</td>
<td>20 $</td>
</tr>
<tr>
<td></td>
<td>Cost of a sink ( (a') )</td>
<td>40000 $</td>
</tr>
<tr>
<td></td>
<td>Battery cost ( (b) )</td>
<td>2 $/J</td>
</tr>
</tbody>
</table>

6.2. Uniform load routing algorithm

In order to balance the burden of sensors on identical layer, a uniform load routing algorithm is proposed in this section. The algorithm determines the next hop sensors of each sensor and the amount of data transmitted to the next hop sensors. The algorithm is detailed as follows. The pseudo code of the ULR algorithm is presented in Table 3.

1. All sensors are divided into six groups, as shown in Fig. 3. Let \( S^j_i \) denotes the jth sensor of i-th layer, \( i = 1, 2 \ldots k \), \( 1 \leq j \leq 6i \).
   The sensors \( S^j_i \) belong to group \( h \) \( (h = 1 \ldots 6) \) while \( i = 1, 2 \ldots k; j(h-1) + 1 \leq j \leq hi \).
2. Next hop sensors of sensor \( S^j_i \) are set. Sensor \( S^j_i \) has one or two next hop sensors. Let \( N(S^j_i) \) denote next hop sensors of sensor \( S^j_i \).

### Fig. 2. A multi-sink network.
that has one next hop sensor. Then, according to the same amount of data that next hop sensors should receive, the ratio of data of \( S_i^j \) that has two next hop sensors is calculated. For example, \( S_i^j \) has one next hop sensor \( S_{i-1}^j \); \( S_i^j \) has two next hop sensors, \( S_{i-1}^j \) and \( S_{i+1}^j \). The ratio is set as 1 for \( S_i^j \). For \( S_i^j \), the ratios are \( \frac{1}{h} - 1 = \frac{1}{r_i^j} \) and \( 2 - \frac{1}{h} = \frac{2}{r_i^j} \). This means \( \frac{1}{r_i^j} \) of all data of \( S_i^j \) is transmitted to \( S_{i-1}^j \), \( \frac{2}{r_i^j} \) of all data to \( S_{i+1}^j \).

(4) From the \( k \)th layer to first layer, each sensor sends data received and generated to the next hop sensors according to the ratio.

### Table 3
The pseudo code of ULR.

```plaintext
Algorithm ULD
for h from 1 to 6
  for each sensor \( C_i^j \)
    if \( i(h - 1) + 1 < j < hi \)
      \( G(C_i^j) = h \)
    end if
  end for
end for
for each sensor \( C_i^j \)
  if \( j = [G(C_i^j) - 1] + 1 \)
    \( N2(C_i^j) = C_i^{h-1}, R2(C_i^j) = -1 \)
  end if
  if \( j = G(C_i^j) \)
    \( N1(C_i^j) = C_i^{h-1}, R1(C_i^j) = -1 \)
  end if
  if \( i(h - 1) + 1 < j < G(C_i^j) \)
    \( N1(C_i^j) = C_i^{h-1}, R1(C_i^j) = -1 - \frac{1}{r_i^j} + R2(C_i^{h-1}) \)
    \( N2(C_i^j) = C_i^{h-1}, R2(C_i^j) = -1 - \frac{1}{r_i^j} + R2(C_i^{h-1}) \)
  end if
end for
for Each sensor from \( k \)th layer to first layer
  Generate and Receive data.
  Send data proportionately to the next hop sensors.
end for
```

6.3. Numerical analysis on energy consumption and cost

In this section, the energy consumption and cost are analyzed numerically without considering constraints (19)-(21). According to the Eq. (4), the Fig. 4 is plotted. It shows the energy consumption of sensor at different layer when the number of layers \( k \) is 7, data compression ratio \( m \) is 0.6, and other parameters are set as Table 2. The energy consumption of sensor at first layer is largest. The energy consumption decreases as layer number increases.

The CPUA is analyzed numerically under the different number of layers and design lifetime of network. Fig. 5 is plotted when the design lifetime is set as \( 6 \times 10^4 \) minutes. Fig. 5 shows that CPUA decreases firstly and then increases when the number of layers increases. The optimal number of layers minimizing CPUA can be found, \( k = 5 \). There is a rapid decline of CPUA when \( k \) changes from 1 to 2. The reasons are as follows. When \( k = 1 \), sink cost accounts for larger proportion in CPUA. When \( k \) changes from 1 to 2, the area of network was increased to 2.7 times, and sink cost in CPUA drops rapidly while energy cost rises less. This makes the CPUA falling fast. Along with increase of \( k \), the increased amplitude of area decreases, and the proportion of the energy cost in the

![Fig. 3. Uniform load routing algorithm.](image)

![Fig. 4. The energy consumption of sensor at different layer.](image)

![Fig. 5. The CPUA under different number of layers.](image)
CPUA increases constantly. So the CPUA slows down or increases.

Fig. 6 shows the CPUA under different number of layers and design lifetime. From Fig. 6, it is observed that the optimal number of layers decreases as the design lifetime increases. It can be explained as follows. The proportion of energy cost in CPUA increases with the increase of design lifetime. In order to minimize the CPUA, the energy consumption is reduced by decreasing the hop number in the process of data transmission. The reduction of hop number means the reduction of $k$.

In order to explore the impact of aggregating data of sink, the variable data compression ratio is used to get CPUA. Fig. 7 shows the relation between data compression ratio and CPUA when $T = 10^5$ minutes and $k = 6$. It is worth mentioning that data aggregation is an efficient method for reducing the CPUA, and improving the compression ratio can linearly reduce CPUA.

6.4. Uniform load routing algorithm and residual energy

In this section, the Uniform Load Routing (ULR) algorithm is simulated. The initial energy of sensors and sink is set according to the above Eq. (4) and (8). Network stops running when residual energy of any sensor is less than a threshold. Because each group runs independently, only one group needs to be simulated. The design lifetime of the network, the number of layers and threshold are set as $10^5$ minutes, 5 and $10^{-3}$J respectively. Fig. 8 shows the residual energy of sensors in the first group. The x-axis represents the layer number. The y-axis represents the sensor number at the same layer, and z-axis represents residual energy. From the Fig. 8, the sensors at the same layer have approximate residual energy. The minimum residual energy is $0.0002$ J that relates to the threshold. Ratio of total residual energy to total initial energy for all sensors and sink is $0.7577$, and the simulative lifetime of network is 24227 min.

6.5. The multi-sink network under constraints

Before deploying multi-sink network, we must get the number of layers $k$ by solving above programming model. As an important
parameter of network, it determines the structure and cost of multi-sink network. The optimal $k$ and CPUA are closely associated with the constraints $(19)$–$(21)$. In this section, we simulate the optimal $k$ and CPUA under the varying numbers of battery levels $M$, maximum buffer size $Max_b$ and transmission range $Ts$ of sink.

It is assumed that $m = 0.6$, $T_s = 10^5$ min, $T_c = 5$ min and other parameters are set as Table 2. The programming model with different values of $M$, $Max_b$ and $Ts$ is solved. The value of $M$ is 3, 4 and 5 respectively. Similarly, $Max_b$ is 30 k, 40 k and 50 k, and $Ts$ is 250 m, 300 m and 350 m. The solutions of model are listed in Table 4. Table 4 shows that under above assumption of parameters, the maximum optimal $k$ is 4, and optimal $k$ decreases as the values of constraints decrease. Because of only 3 values for each constraint, above conclusion is simple. In order to analyze in detail the relationship between optimal $k$ and each constraint, we fix the values of two constraints and then get the optimal $k$ corresponding to value of third constraint that varies in big range by solving the model respectively. Fig. 10–12 are plotted. We can explain the Figures from the viewpoint of Operational Research. The three constraints determine the value of $b$ that is the upper bound of the feasible region $[1, b]_o$ of $k$. The optimal $k$ is 4 without constraints under assumption of parameters. When $M = 5$, $Max_b = 50$ kb and $Ts > 260$ m, three constraints are inactive constraints and $4 \in [1, b]$, so the optimal $k$ is 4 (see Fig. 10). With the decrease of $Ts$, $b$ decreases (means to narrow the feasible region), and the optimal $k$ decreases while the $Ts$ constraint becomes an active constraint. Fig. 11 and 12 have similar explanations. It is also concluded that larger values of $M$, $Ts$ and $Max_b$ will not change the optimal $k$.

It is noted that not all the subnetworks consist of optimal number of layers due to the size or shape of monitored area. One or several subnetworks possibly have fewer layers. A sample is given in the next section.

6.6. Comparison of single-sink network and multi-sink network

In order to minimize the network cost, several means are used in the deployment strategy for a multi-sink WSN, such as regular

<table>
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<tr>
<th>$M$</th>
<th>$Max_b$(kb)</th>
<th>$Ts$(m)</th>
<th>The optimal $k$</th>
<th>CPUA ($)</th>
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</table>

Fig. 10. The optimal $k$ at different $Ts$.

Fig. 11. The optimal $k$ at different $Max_b$.

Fig. 12. The optimal $k$ at different $M$. 
hexagonal cell architecture, different levels of battery, and optimal layer number. In this section, we compare the cost of single-sink network and multi-sink network under different condition. The design lifetime of the network is set as $6 \times 10^3$ minutes. For simplicity, we assume that buffer size and transmission range of sink can meet the single-sink network requirements and the number of battery levels is large enough. Two network structures are set up, a single-sink network and a three-sink network. The single-sink network consists of nine layers, 270 sensors. In the three-sink network, each subnetwork consists of five layers (optimal layer number), 90 sensors. The area monitored by two networks is approximately same. The CPUA of networks is simulated under two conditions (different levels of battery or same battery are equipped to sensors). The three-sink network with different levels of battery corresponds with our deployment strategy. Table 5 gives the CPUA of four networks. From the table, it is showed that the two means are efficacious in reducing CPUA of network. It is also proved that our strategy is effective at minimizing cost of network.

Furthermore, another group of networks are considered. A single-sink network consists of ten layers, 330 sensors; a multi-sink network consists of three subnetworks of five layers and one sub-network of four layers. As a result of the limitation of monitored area, the multi-sink network is a suboptimal network structure. Same as above simulation, we can get the CPUA of networks under two conditions. Similar conclusions are obtained.

We have not compared proposed strategy to the other existing methods. The main reason is as follows. The existing methods decrease energy consumption or cost from unilateral, such as only RN placement, or energy balance, etc. But, the strategy proposed in this article covers multiple aspects, the number and location of SN and sink, energy distribution, and optimal layer, data routing, etc. From the perspective of minimizing total cost of network, there are few comparability between them.

7. Conclusion

In this paper, a deployment strategy of WSN is proposed based on minimizing cost per unit area. Regular hexagonal cell architecture is employed to satisfy the constraints of coverage and connectivity. Then, the energy consumption of sensors and sink and cost of network are explored by theoretic and numerical analysis. An Energy Allocation Theorem and an Integer Programming model are presented to minimize the CPUA. Furthermore, we propose a scheme of multi-sink network. In the simulation phase, a Uniform Load Routing algorithm is designed to balance the energy consumption of sensors on the identical layer. Simulation results show that our scheme significantly reduces the cost of WSN.

Future extensions of this work can be done in two directions. First, in this study, it is assumed that the scale that needs to be monitored can be divided into regular hexagons of radius $R_h$. However, such cases are rare in practice, therefore, finding suboptimal partition (corresponding to suboptimal cost) is an interesting task. Second, because of the limits of energy-block type, the configuration of initial energy is not arbitrary. How to pack energy-block to fit initial energy is another problem related to cost of WSN.

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### References


### Table 5

<table>
<thead>
<tr>
<th>CPUA ($)</th>
<th>Multi-Sink WSN</th>
<th>Single-Sink WSN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Different levels of battery</td>
<td>1.6142</td>
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<tr>
<td>Same battery</td>
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